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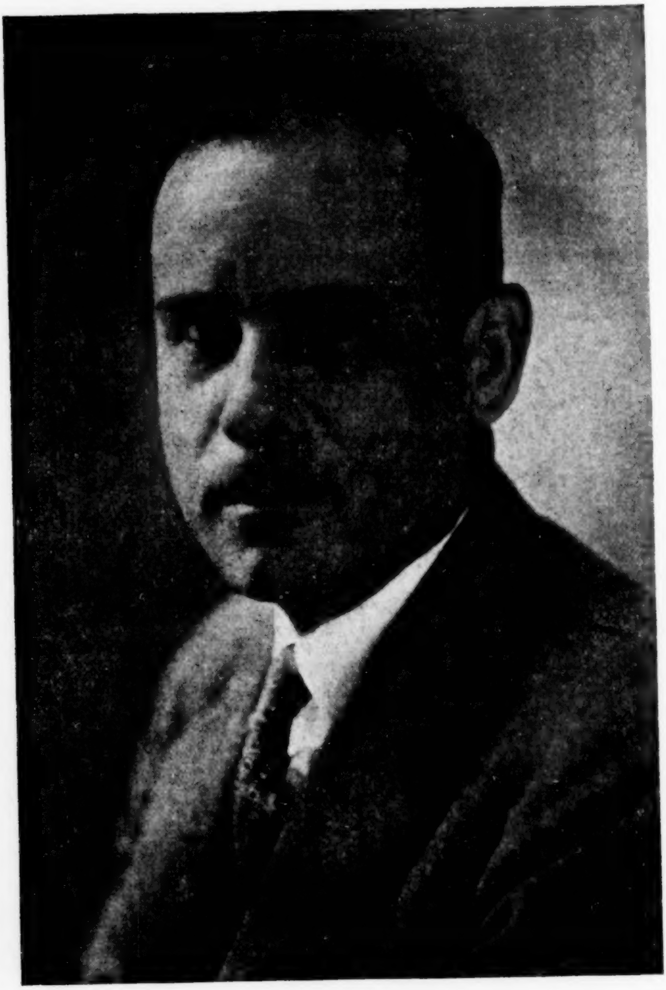

J. O. HASSLER

# THE MATHEMATICS TEACHER

Volume XXVII Number 5

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## A Message From The New President

IN ASSUMING THE DUTIES of the office of president of the National Council I must confess to a feeling of apprehension because of the responsibility entailed. During the fourteen years of the Council's existence a high level of achievement has been set by its seven able presidents, aided by thirty-five other teachers serving in various official capacities and by numerous contributers to the *Mathematics Teacher* and the yearbooks. To direct the activities of the Council in such a manner as to maintain the present high standards is a task that presents a challenge to any man. It cannot be done by one person alone. For the work of the Council to be a success there must be cooperation.

The function of the president is to direct the work on policies adopted by the Council, propose new policies for adoption, and unify and harmonize the activities of the various officers and committees. To that task I pledge for the term of my office the best service consonant with my ability and ask the faithful cooperation of every member and officer.

I am convinced that considerable influence on American education has been exercised by the Council. Coming into existence at about the time of the preparation and publication of the epochmaking *Report of the National Committee,* the Council has done much to further the ideals there set forth. It has been directly responsible for the publication in the last decade of contributions to

the literature on the teaching of mathematics which are worthy successors to that famous *Report* and which rank second to nothing else that has ever been presented in the United States.

All of this avails us nothing if the teachers of secondary mathematics do not read what is produced. It is lamentable that the major part of those who are teaching mathematics in our high schools today are not familiar with our publications. Many do not know that there is such an organization as the National Council and have never heard of the *Report of the National Committee.*

Such a situation furnishes a job for every member of the Council. Each can have a share in improving the teaching of mathematics in the United States. The writers have labored long *for no monetary reward* to produce the material. It is our job to get it to the consumer. No state or sectional meeting of mathematics teachers should be held without some member presenting the cause of the Council and securing memberships. Let your salutation to fellow teachers be, "Are you a member of the National Council?"

Much has been said and written about mathematics being in bad repute. The wonder is that under the type of teaching to which mathematics has been subjected, it even remains in the curriculum. Loose-thinking educational administrators blame the subject itself and would cast it out. Our first reaction is to rush to its defense with flowery encomiums setting forth its wonderful value and usefulness to mankind. It is my opinion that rather than "saying it with flowers" we should "say it with improved teaching." Instead of parading the virtues of mathematics on paper and in speeches, let us teach its values into the lives of our pupils and it will need no defense.

As a body of teachers we cannot do this if most of us know little or nothing about improved methods of teaching and less about the subject itself. It is the job of some of the members of the Council to produce the material for the education of the rank and file in both *subject matter* and *methods* and the job of all of us to see that this material is spread from coast to coast. Let every member choose his task and never cease to work for the advancement of the Council, which means the advancement of the educational standing of our chosen subject, mathematics.

J. O. HASSLER, *Norman, Okla.*

# Development of Mathematics in Secondary Schools of the United States

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PART III

## MATHEMATICS DURING THE HIGH SCHOOL PERIOD*

THE CHANGING social ideals and demands, which had caused the downfall of the Latin grammar school and had given impetus to the growth of the academy, soon began to uncover the shortcomings of this new form of secondary education. The academy was largely privately owned and operated and consequently was somewhat dependent upon tuition fees, a fact which, to a very considerable extent, limited the service rendered. Public sentiment soon began to demand a form of education that would more nearly reach the masses. This meant an educational organization supported and controlled by the people, that is, supported by public taxation and under state supervision. The old European idea that instruction beyond the "three R's" was meant only for the aristocracy seemed to predominate in the educational philosophy of the average American citizen of this era and he was rather slow to accept the scheme of free public secondary education on a national plan. The first school organized under this new plan was the "English Classical School" established in Boston in 1821, the name being changed to the "English High School" in 1824. The purpose of this school was to offer a finishing course for boys preparing for mercantile or mechanical activities, and the course of study was very similar to the one offered at this same time in the English Department of the Phillips Exeter Academy.[1] It was not until the last

* This is the third part of a discussion on the "Development of Mathematics in Secondary Schools of the United States." The second part appeared in the April issue of THE MATHEMATICS TEACHER. The fourth and last part will appear in the October issue.

[1] E. E. Brown, "Secondary Education," Annual Report of the Department of Interior, Bureau of Education, Washington, D. C., (1903), Vol. I, p. 563.

quarter of the nineteenth century that the high school predominated over the academy. Rapid changes were being made in the social, political, and industrial life of America and evidence of the attempts of the high school to keep pace with these developments may be seen in the large number of courses added to the curriculum. Increased interest in high school education brought about an increased interest in going to college. The high school, however, soon realized that it had other duties and responsibilities than that of preparing its students for college and this attitude on the part of the secondary schools was instrumental in bringing about changes in college entrance requirements. Mathematics was commonly required but in varying degrees. An examination of the catalogues of eleven representative colleges in 1870 revealed that only two required algebra beyond quadratic equations, the usual requirement being to quadratics, and in geometry there were only two that required as much as all of plane.[2] Twenty-six years later the report of the Commissioner of Education printed the following analysis of the entrance requirements in mathematics as found in the catalogues of 432 institutions:

> With respect to the requirements in mathematics, it may be seen that 346 institutions require an examination in arithmetic. The other 86 institutions that do not require an examination undoubtedly suppose that the study of arithmetic has been completed before a candidate presents himself for admittance to college. The requirements in arithmetic usually include a knowledge of the metric system of weights and measures. Algebra is required by 412 institutions, leaving but 20 institutions that admit students to an A.B. course without some knowledge of algebra. Algebra, through quadratics or beyond, is required by 174 institutions; to quadratics, is required by 37, and 201 institutions simply state that algebra is required. With respect to geometry, it is found that 294 institutions require plane geometry; 93, solid geometry, and 8, spherical geometry. Two institutions require an examination in conic sections, and 4 in trigonometry.[3]

After 1890 there seemed to be a trend toward the requirement of two units in secondary mathematics, one in algebra and one in geometry. In the period from 1913 to 1922 all colleges and universities, except Leland Stanford and the University of Chicago, required some form of mathematics for entrance. Those requiring two units increased during this period from 21 per cent to 48 per

[2] E. C. Broome, A Historical and Critical Discussion of College Admission Requirements, The Macmillan Co., New York, (1903), p. 67.

[3] Annual Report of the Department of the Interior, Bureau of Education, Washington, D. C., (1896–97), Vol. I, p. 468.

cent, while those requiring two and one half units decreased from 53 per cent to 33 per cent, and those requiring as much as three units, from 21 per cent to 11 per cent. The requirements in algebra and plane geometry changed but very little while arithmetic, trigonometry, and solid geometry practically disappeared from the list.[4] The partial returns from an investigation of the entrance requirements of forty-six state-owned and forty-nine non-state-owned colleges and universities revealed that, in 1923, eleven institutions did not require any mathematics for entrance to the Liberal Arts course, forty required two units, while twenty-seven specified two and one-half. In the same group for 1928 there were eight institutions that required no mathematics, forty-two two units, and twenty-two two and one-half. In 1923 candidates for admission to curricula leading to the Bachelor of Science degree were required to present no credit in mathematics by three institutions, two units by thirteen, two and one-half by eight, three by nine, and four by two; while in 1928 there were only two institutions requiring no entrance credit in mathematics, two that required one unit, seventeen two units, seven two and one-half units, four three units, two three and one-half units, and only one that required four units.[5]

We have seen how mathematics was introduced into the curricula of the Latin grammar schools and how it also occupied a prominent place in the program of the academy. It was well represented in the outline of the first course of study in the English High School in 1823.[6] Although only three towns in the state of Massachusetts claimed to offer algebra in 1834, it was listed in the courses of one hundred towns in 1840–41, the study even being

[4] H. C. McKown, "Trend of College Entrance Requirements," Bulletin #35, Bureau of Education, Department of Interior, Washington, D. C., (1924), pp. 64–66.

[5] S. W. Williams, Trends in Mathematical Requirements for the Bachelor's Degree, Unpublished Master of Arts Thesis, George Peabody College for Teachers, (1928), pp. 19–20, 23.

[6] The outline as given by L. V. Koos, The American Secondary School, pp. 31–22, shows the following mathematics program: *3d or Lowest Class*—Intellectual and Written Arithmetic by Colburn and Lacroix, Bookkeeping by Single and Double Entry. *2d Class:*—Arithmetic and Bookkeeping continued, Algebra by dictation and Colburn, Geometry by Legendre. *1st Class:*—Algebra and Geometry continued, Practical Mathematics, Comprehending Navigation, Surveying, Mensuration, Astronomical Calculations, etc., together with the construction and use of mathmatical instruments.

extended down into the elementary school in a few cases. While only two towns in the same state claimed to offer geometry in 1834, this number had increased to eighteen by 1840. By 1860 both algebra and geometry had become firmly established in the curricula of the state high schools and at this same time we find that forty-five of the sixty-three state schools offered surveying; fifteen offered navigation, mensuration, and trigonometry; and one school listed analytic geometry.[7] Mathematics was not the only subject to attain or hold a place of prominence in the high school curriculum. As in the academy there was a strong tendency toward expansion both in number and content of courses offered, as well as an attempt to blend intellectual and practical training in the same school. Curricula were organized and expanded rapidly with no particular plan or definite educational objective in view. By 1890 this unrest had reached its highest point.[8] This awkward and unsystematic expansion of the curriculum offered sufficient reason for a demand for reform and mathematics received its share of the attack. David Eugene Smith was positive in his contention that in neither algebra nor geometry "did there seem to be any clear conception of the purpose of teaching mathematics in the twentieth century as distinguished from that which came into being with the rise of analysis and algebraic symbolism three hundred years ago."[9] Dissatisfaction had arisen from several sources relative to the results achieved in the teaching of secondary mathematics. Complaints had come from the teachers of mathematics themselves that the subject was not being grasped by the pupils. A study of a large number of representative high schools revealed that the largest percentages of failures were in Latin and mathematics. Nineteen and five-tenths per cent of all pupils enrolled in mathematics in the several high schools failed to pass.[10] College faculties were not hesitant in letting it be known that students entered their freshman classes with poor mathematical training. Business men were doubtful of

[7] A. J. Inglis, Rise of the High School in Massachusetts, Columbia University Press, New York, (1911), pp. 111–115.

[8] I. L. Kandel, History of Secondary Education, Houghton Mifflin Co., New York, (1930), p. 461.

[9] D. E. Smith, "A General Survey of the Progress of Mathematics in our High Schools in the Last Twenty-five Years," First Yearbook of the National Council of Teachers of Mathematics, Columbia University Press, New York, (1926), p. 3.

[10] F. P. O'Brien, The High School Failures, Columbia University Press, New York, (1919), p. 21.

the opportunity for the application of high school mathematics, as taught, to problems of everyday life. Secondary mathematics had been weighed in the scales of "results produced" and had been found wanting.

The Committee of Ten on Secondary Education, sponsored by the National Educational Association agreed that a radical change in the teaching of mathematics was necessary. The Subcomittee on mathematics recommended that a course in concrete geometry with numerous exercises be introduced into the grammar school, and that systematic algebra should begin at the age of fourteen.It was suggested that demonstrative geometry be begun at the end of the first year in algebra and be taught along with algebra for the next two years and that work in solid geometry might be incorporated. Formal algebra was to be studied for five hours a week during the first year and for two and one-half hours a week for the two following years, during which time it was to parallel work in geometry. Special emphasis was to be placed on literal as well as numerical coefficients. The Committee also suggested that those who did not expect to go to college might, after the first year of algebra, turn to bookkeeping and the technical parts of arithmetic, while boys planning to attend scientific school might profitably spend a year on trigonometry and some more advanced topics of algebra. A hope was expressed by the Committee that a place might be found in the high school or college course for at least the essentials of modern synthetic or projective geometry.[11]

The question still remained as to how colleges could best adjust their requirements to meet these changing conditions in the secondary schools. This question was partially answered by the Committee on College Entrance Requirements which made its report to the National Educational Association in 1899. This Committee made recommendations for the high school course in mathematics which were quite in accordance with the previous recommendations of the Committee of Ten. It was urged that no college require for entrance any more than algebra and plane geometry. The following course was suggested:

*Seventh Grade: Concrete Geometry and Introduction to Algebra. (four periods); EighthGrade: Introduction to DemonstrativeGeometry, Algebra, (four periods); Ninth and Tenth Grades: Algebra and Plane*

[11] Report of the Committee of Ten on Secondary School Studies, American Book Co., New York, (1894), pp. 105–107.

*Geometry, (four periods); Eleventh Grade: Solid Geometry and Plane Trigonometry, (four periods); Twelfth Grade: Advanced Algebra and Mathematics reviewed, (four periods).*

Algebra for the seventh grade was to begin with literal arithmetic which was to be followed by simple polynomials and fractional expressions, equations of the first degree, the four fundamental operations for rational algebraic expressions, simple factoring, and solution of equations of the first degree in one and two unknowns. One half of the time in the eighth grade was to be spent in demonstrrative geometry which was to include the study of congruent triangles, parallel lines, sum of the angles of a triangle, parallelograms, and the more useful properties of the circle. Ninth grade algebra was to include: (a) a more systematic and comprehensive study of previous work; (b) radicals, fractional exponents, and quadratic equations in one and two unknowns; (c) ratio and proportion, the progressions, and an elementary treatment of permutations and combinations; and (d) the binomial theorem for positive, integral exponents, and logarithms. Advanced algebra was to give an elementary treatment of infinite series, undetermined coefficients the binomial theorem for negative and fractional exponents, determinants, and the theory of equations. Solid geometry and plane trigonometry were to be essentially the same as the college courses in those subjects. The following credit distribution was recommended for college entrance: Elementary Algebra, $1\frac{1}{2}$; Advanced Algebra, $\frac{1}{2}$; Plane Geometry, 1; Solid Geometry, $\frac{1}{2}$ and Plane Trigonometry, $\frac{1}{2}$.[12]

At the same time that these national committees were working toward standardization of mathematics courses in the secondary schools and the closer coordination between high schools and colleges in entrance requirements, other influences were striving for improvement in methods of mathematical instruction. Probably the first purposive effort directed toward the improvement of teaching of secondary mathematics was that which had its beginning in England in the so-called Perry Movement. Professor John Perry, in his address before the first meeting of the Section of Educational Science at the Glasgow meeting of the British Association, in 1902, contended that mathematics affected less than one per cent of the boys who were supposed to study it, and complained

[12] "Report of the Committee on College Entrance Requirements" Annual Report of the National Education Association, (1899), pp. 627, 649–651.

that "We teach all boys what is called mathematical philosophy, that we may catch in our net that one demigod, the one pure mathematician, and we do our best to ruin all of the others."[13] Perry could not see the harm in letting a pupil assume the truth of many propositions of the first four books of Euclid, in letting him accept the truth partly by faith and partly by trial, or in giving him the whole fifth book by algebra. He believed that teachers of demonstrative geometry and orthodox mathematicians in general were not only destroying what power to think already existed, but were producing a dislike, a hatred for all kinds of computation and therefore for all scientific study of nature, and were thus doing an incalculable harm.[14] This address, quite radical in its contentions, brought about a slow change in the teaching of mathematics in England. Throughout England today two courses in mathematics are offered, one styled "pure mathematics" and the other "practical mathematics."

Contemporaneous with the Perry movement in England there was a similar era of progress, or change, in America which was set in motion by the address, in 1902, of Professor E. H. Moore before the American Mathematical Society. Moore, like Perry, felt that, by lessening the emphasis on the systematic and formal side of instruction in mathematics on the one hand, and by increasing the emphasis on the practical side of the subject matter and its relation to physics, chemistry, and engineering on the other, it would be possible to give very young students an interesting introduction to the essential notions of trigonometry, analytic geometry, and calculus. He argued that a student, by making use of the "finest mathematical tools which the centuries had forged" in scientific investigations, would come to be interested not only in the achievement but in the theory of the tools themselves. The fundamental problem, as he saw it, was the unification of pure and applied mathematics and the correlation of the different subjects of the curriculum.[15] Although the ideas expressed by Moore were not accepted as a whole by all persons interested in the pedagogy of mathematics,

[13] John Perry, "Teaching of Mathematics," Educational Review Vol. 23, (1902), p. 163.

[14] John Perry, *ibid.*, pp. 169–172.

[15] E. H. Moore, "Foundations of Mathematics," First Yearbook of the National Council of Teachers of Mathematics, Columbia University Press, New York, (1926), pp. 42–47.

his address did stimulate the teaching of intuitive geometry in this country, it also started a reaction to the strict formalism found in algebra.

Another influence contributing to the improvement in the teaching of secondary mathematics was that due to the reports of the International Commission on the Teaching of Mathematics which were published by the United States Bureau of Education between the years 1911 and 1918. This Committee found that every high school offered algebra and geometry for at least one year each. One-half of the schools gave algebra for an extra half-year and less than twenty per cent gave algebra for the full two years. There were very few schools that offered algebra for two and one-half years and only the larger high schools had courses in solid geometry, plane trigonometry, and advanced algebra. Algebra I consisted of: (1) four fundamental operations; (2) factoring; (3) highest common factor and lowest common multiple; (3) fractions of ingenious complexity; (5) simple equations and problems; (6) linear eliminations and problems; (7) minor treatment in practice of deriving equations; (8) involution and evolution; (9) radicals and radical equations; and (10) exponents with no mention of logarithms. The course in Algebra II included: (1) quadratics in one and two unknowns; (2) ratio and proportion; (3) progressions; (4) inserting means; and (5) binomial theorem.[16]

The sequence in all of the ordinary textbooks in geometry was that of Legendre. Attempts were made to master all of the usual cases of incommensurables. Geometrical constructions by the Euclidean method were usually given a logical place among other propositions. The original exercises, which ranged from six hundred in one text to twelve hundred in another were rigidly confined to the subject matter of the text. The order of development in solid geometry included successively the following topics: (1) perpendicular and parallel lines and planes; (2) dihedral, trihedral, and polyhedral angles; (3) equivalence and congruence of parallelopipeds, prisms, pyramids, cylinders, and cones; (4) geometry of the great-circle diagrams on a spherical surface; and (5) the surface and volume of a sphere.[17]

[16] "Report of the International Commission on the Teaching of Mathematics in the Public and Private Secondary Schools of the United States," Department of Interior, Bureau of Education Bulletin, Government Printing Office, Washington, D. C., (1911), Vol. 16, pp. 17–20.

The study of trigonometry involved (1) the function of an acute angle and the change in value of one function from one quadrant to another; (2) proof of the formulae for the sine and cosine of the sum and difference of two angles, of the tangent of the sum and difference, of the sine, cosine, and tangent of the double and the half-angle; (3) formulae for the transformation of the sum or the difference of two sines or two cosines into products; (4) practice in the use of all three formulae in reduction; (5) general logarithms; and (6) application of logarithms to the solving of triangles, and problems in surveying and navigation.[18]

Under the title of Advanced Algebra various topics were included as follows: (1) theory of quadratic equations with graphs; (2) solution of numerical equations of more than one unknown; (3) occasional trigonometric solutions; (4) choice and chance; (5) determinants with practice in reduction; and (6) undetermined coefficients. In addition to subjects usually recognized as secondary school subjects, several high schools offered a course in commercial arithmetic similar to the course frequently given in the elementary school except that the problems were somewhat more difficult and more closely related to commercial life.[19]

The Committee also made a study of the qualifications and training of teachers of secondary mathematics in the United States. The existent situation was far from satisfactory for not only did they find that the large majority of all teachers were very poorly prepared but also that there were so few institutions of higher learning equipped for the training of teachers of mathematics that no real constructive program of preparation could be put into effect.

The benefits accruing from the work of this committee may be rather vividly summarized in the recommendations which it made for a constructive program for improving the teaching of mathematics in the secondary schools. These may be grouped under two classifications:

I. Program for the training of teachers.
   1. Trigonometry, college algebra, analytic geometry.
   2. Surveying, or descriptive geometry, or elementary astronomy.
   3. Differential and integral calculus with applications to geometry, mechanics, and physics.

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[17] *Ibid.*, p. 20.

[18] *Ibid.*, p. 21.

[19] *Ibid.*, pp. 21–22.

4. Modern geometry.
5. Elements of analytic mechanics.
6. Elements of theoretical and laboratory physics.
7. Algebra from the modern standpoint.
8. Course in history of mathematics.
9. Course in the teaching of mathematics.

II. Improvement of curriculum and method.
1. Change in aim.
   a. To modify to conform to what is understood to be the outcome of recent psychological research concerning the value of formal discipline.
   b. Tendency to attach greater importance to utilitarian mathematics.
2. To omit geometry proofs that are too difficult or too obvious.
3. To avoid algebraic manipulations of greater complexity than is requisite to prepare thoroughly for the work that lies beyond.
4. To give more prominence to the equation.
5. To introduce more problems from physics, other sciences and practical life.
6. To transfer more difficult algebraic matter to later years in High School.[20]

These recommendations along with those suggested by the Committee of Ten, the Committee on College Entrance Credits, and by Professors Perry, Nunn, Klein, and Moore served to introduce changes in the mathematics curriculum and in the methods of instruction on the secondary school level. Contemporaneous with these innovations there was a new movement taking form that was later to revolutionize the organization of the entire High School program. Up to this time all changes had been of a more or less horizontal character, that is no question had been raised as to the desirability of readjusting the time devoted to secondary education, and all recommendations for improvement were directed toward readjustments within the allotted four years. The underlying philosophy of this new movement in education was that there should be a more equal division of time between elementary and secondary education, and that the pupils of grades 7–8–9 formed a more or less homogeneous group that had interests and presented problems distinct in their nature from those of either the earlier elementary grades or the later years of the high school. This vertical expansion in secondary education presented new problems in matters of curriculum and methods of teaching in secondary school mathematics, the full significance of which may be seen only by a careful study of the trends of mathematical instruction in the junior and senior high schools.

[20] "Report of the American Committee of the International Commission on the Teaching of Mathematics," Bureau of Education Bulletin, Department of Interior, Washington, D. C., (1912), Vol. 14, pp. 33–38.

# A Study of Prognosis of Probable Success in Algebra and in Geometry (continued)[1]

*By* JOSEPH B. ORLEANS
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## SOURCES OF DATA

THE SPECIFIC ABILITY TEST in algebra[2] used in this study consists of a short preliminary part of thirteen questions in arithmetic, eleven short lessons, each followed by an exercise based on the lesson, and a summary test at the end. The topics covered in these lessons are the use of letters for numbers, the use and the meaning of the exponent, like and unlike terms, algebraic representation, signed numbers and the solution of simple problems.

The specific ability test in geometry[3] consists of nine lessons, each followed by an exercise based on the lesson, and a summary test at the end. The topics covered are the meaning and the applications of the axioms, the use of common terms like acute, obtuse, right, and straight, supplementary, complementary, vertical angles, diagonal, bisect, the reading and interpretation of diagrams, recognizing hypothesis and conclusion, and simple proofs.

The test was given each time before the pupils began the study of the subject. The total time for the administering of the algebra test was eighty-one minutes, a definite number of minutes being assigned to each part, plus nine minutes for passing out the booklets, for filling in the information on the front page and for the preliminary instructions. The test was given in either one or two sittings, depending upon the individual school. In the latter case, the first sitting ended with part 7. Detailed instructions accompanied the tests, so that the administering of the test might be as uniform as possible.

For the geometry test the total time was eighty minutes which included ten minutes for the preliminary preparation. It was also accompanied by specific printed directions for the administering of the test. Directions for scoring the tests and printed keys were furnished. The scoring was, therefore, uniform and objective.

[1] This is the second part of this study. The first part appeared in the April number of THE MATHEMATICS TEACHER.

[2] The Orleans Algebra Prognosis Test. World Book Co. 1928.

[3] The Orleans Geometry Prognosis Test. World Book Co. 1929.

The achievement test used at the end of the first term of the algebra was the Columbia Research Bureau Algebra Test I, consisting of 36 questions in Part I (Mechanics) and 18 questions in Part II (Problems). The questions cover the work that is ordinarily taught in the first term of the algebra of the ninth year.

The achievement test at the end of one term of geometry was the Orleans Plane Geometry Achievement Test I, an eight page booklet covering the work that one finds in Books I and II of the usual plane geometry textbook, excepting the work on loci and the constructions at the end of Book II.

To compute the reliability of the prognosis tests, since it was not possible to repeat them or to give the same group of pupils two forms, the writer obtained the correlation between the scores on the odd and the even numbered questions in the exercises and then applied the Spearman-Brown formula

$$r_{\text{II}} = \frac{2\,r_{o.e.}}{1 + r_{o.e.}}$$

The correlations obtained were .93 and .96, for the algebra and geometry respectively, the number of cases being 220 and 120. For the achievement tests the coefficient of reliability was obtained by correlating Form A with Form B. The correlations obtained were .89 and .94 for the algebra (147 and 115 cases respectively) and .85 for the geometry (625 cases). In order to understand the significance of these values, one must remember that "reliability refers to the degree to which the test measures whatever it does measure or to the amount of confidence that may be placed in the mark or score as a measure of some ability of the pupil."[4]

The validity of the achievement tests, indicated by the correlations between the test scores and teacher's ratings, is represented by the following data: for four groups in algebra, the number of cases being 53, 95, 38, 67, the correlations varied from .80 to .89; for fourteen groups in geometry, the number of cases ranging from 28 to 93, the correlations varied from .73 to .96. "Validity is the degree to which a test measures what it is intended to measure. It is in general the degree to which a test parallels the curriculum and good teaching practice."[5] The correlations obtained are about as high as the unreliability of teachers' marks will permit.

[4] Ruch, G. M. The Objective or New Type Examination. Scott Foresman and Co., 1929.

[5] *Ibid.*, pp. 27, 28.

The groups to which the algebra tests were given consisted of boys and girls in the first year of the Senior High School. The pupils who took the geometry tests were in the tenth grade, some in the first half, others in the second half, most of the former having been admitted from the Junior High Schools. All the pupils began the study of geometry immediately after completing one year of algebra. The groups from all the schools but one, the De Witt Clinton High School, which is a boys school, consisted of boys and girls. They were not selected groups. Pupils had been assigned to the classes at random in accordance with the method of program making in vogue in the schools. The accompanying tables give the age distribution of some of the groups and the distribution of the I.Q.'s obtained from the Otis Self-Administering Intelligence Test.

*Age Distribution of Pupils Who Took the Algebra and Geometry Prognosis Tests*

| | N | Range | Mean Age | $\sigma$ |
|---|---|---|---|---|
| Algebra | 141 | 11–16 | 14.28 | .92 |
| | 297 | 12–16 | 14.92 | 1.03 |
| Geometry | 535 | 11–18 | 13.57 | 1.26 |
| | 151 | 12–17 | 13.91 | 1.12 |

*Distribution of Otis I.Q.'s of Pupils Who Took the Algebra and Geometry Prognosis Tests*

| | N | Range | Mean I.Q. | $\sigma$ |
|---|---|---|---|---|
| Algebra | 220 | 80–132 | 107.14 | 10.09 |
| Geometry | 235 | 77–136 | 112.56 | 9.56 |

After the tests had been administered several times, the question arose as to whether every part of each test was essential for prognosticating success or failure in algebra or geometry. The correlations were, therefore, computed between each part of the test and marks in achievement at the end of the term. From an examination of the results it seemed that parts arithmetic and 6 in the algebra test and parts 1 and 8 in the geometry test show the poorest correlation with achievement and, therefore, might have been omitted. It then occurred to the writer that possibly this might be a way of shortening the tests. The practical administrator is always faced with the time element in school matters. Teachers feel that the number of days for teaching the course is too small and the

frequent interruptions too many. The parts, therefore, that do not aid in the prediction might easily be dispensed with. Using a coefficient of .35 as the dividing line, the writer decided to disregard parts arithmetic, 3 and 6 of the algebra test and parts 1, 3 and 8 of the geometry test by deducting the scores on these parts from the total scores and computing the correlations of the reduced scores with the marks for achievement. The differences between the correlations with the original scores and those with the reduced scores were not as great in the algebra as in the geometry. It is quite possible, therefore, that the algebra test might be shortened by the omission of parts arithmetic, 3 and 6. In the case of the geometry the variation between the two sets of coefficients of correlation is not sufficiently uniform and it would not be safe to reach a conclusion without a more exhaustive study of the question.

It is interesting to note the content of the parts that have the lowest correlation with achievement. Teachers of experience have generally believed that a mastery of the fundamentals in arithmetic is an essential prerequisite for the study of algebra; and yet the correlation between the arithmetic part of the prognosis test and achievement in algebra is about .27. Later on in this report the reader will find data showing that the correlation between elementary school arithmetic marks and algebra marks is not of much greater significance. It seems, then, that although fundamental arithmetic skills are necessary, there are other factors that are of greater importance.

The extremely low correlation of .17 between part 6 and algebra marks indicates that the lesson on like and unlike terms can certainly be omitted, since the ability to master the definition seems to tell practically nothing about success in algebra. It is possible that a lesson dealing with any other definition would have given a similar result. This seems to be borne out also by the low correlation between geometry marks and scores on that part of the geometry test in which the pupils are taught the meaning of the terms hypothesis and conclusion, and also by the correlation with part 3 which deals with the names of the various kinds of angles.

The possible omission of part 3 of the algebra test is justified by the intercorrelations of the various parts. Of the four parts that involve the use of exponents, namely, 2, 3, 4 and 5, lesson 3 had the lowest correlation with marks in algebra. Since the correlation

between lessons 3 and 4 is .5007, there would be no loss in omitting lesson 3.

## The Data

The algebra prognosis test was tried for the first time in mimeographed form in the fall of 1926 with 300 students taught by two teachers in the George Washington High School, New York City. At the end of that term the pupils were given an objective achievement test on the work of the term. The teachers ratings were also listed. The correlation between the prognosis test scores and the average of the teachers' ratings and the objective test scores was $.635 \pm .035$ for teacher *A* and $\pm .435.046$ for teacher *B*. The partial regression equation was set up, furnishing optimum weights to be assigned to the parts of the test. The multiple correlations of $.80 \pm .021$ and $.60 \pm .036$ respectively were then obtained. These coefficients suggested the correlation which might be expected between observed test scores and marks of achievement, as predicted by this regression equation.

Since this was the first attempt to experiment with the test in its original form, the author gave it also to the classes taught by two poor teachers *C* and *D*, in order to see the effect of the teacher factor on the results. It is interesting to note that the highest correlation was obtained with the group taught by the teacher who was considered by her supervisor to be the best of the four. Low correlations were found in groups taught by teachers *C* and *D*, even though the four groups did equally well in the prognosis test.

The prognosis test was then revised somewhat and used again in the fall of 1927 with 130 pupils taught by one teacher in the De Witt Clinton High School and 120 pupils taught by one teacher in the James Monroe High School, both in New York City. The criterion at the end of the term was the same objective test that had been used the preceding year. For the second group the New York Rating Scale of School Habits was filled in, as described earlier in this report, by the three teachers who were teaching those pupils the subject other than algebra during the term. The sum of the three ratings was listed as each pupil's score on the Rating Scale. The coefficients of correlation between the scores on the prognosis test and the marks on the objective test was $.71 \pm .03$ (for De Witt Clinton) and $.68 \pm .03$ (for Monroe). The correlation between the

prognosis test scores and the objective test marks and rating scale scores for the Monroe group was $.75 \pm .03$.

There are a number of factors that influence correlations like the above, but which can be controlled only in part, if at all. Such factors are the effect, on the pupil's work, of absence, poor physical condition, incompatibility with the teacher, the number of hours spent outside of school on remunerative work, home facilities for study, and so on. It is hardly possible that a teacher, even an excellent one, will as a result of a half year's effort get each pupil to do as much work as he should. The differential between the pupil's achievement under normal conditions and what he could achieve under unusually good conditions would not be the same for all pupils; and this variation might seriously affect the correlations given above. An inadequate criterion will also reduce the correlation by not differentiating adequately between the achievement of the pupils. All these factors, as well as others, have the effect of lowering the correlations computed. The use of the rating scale together with the prognosis test would enhance its prognostic value.

It should also be kept in mind in interpreting the data that the scores obtained by the pupils on the achievement test in algebra were taken as the criterion against which to validate the prognosis test. The scores on the achievement test were taken as measures of achievement of the pupils in the work of the half year. To the extent to which the achievement test was inadequate for this purpose, the prognosis test was not valid. As it turned out, the average score on the achievement test was rather low, being only about thirty per cent of the total possible score on the test. This was due to the fact that the test was evidently too difficult, since none of the pupils attained scores above twenty-five out of a maximum of thirty-six. The spread, however, between the lowest and the highest scores attained was satisfactory. Had a more comprehensive achievement test been used, the correlations would doubtless have been appreciably higher.

The prognosis test was again revised by the addition of Lesson and Test 11 and Test 12 and by the lengthening of the other parts. The objective test was also revised and put into its present form.

The new form of the prognosis test was given in September 1928, February and September 1929 and February 1930 to groups of pupils in the George Washington, the James Monroe, the Bryant

and the Thomas Jefferson High Schools, New York City, in the Dickinson High School, Jersey City and also in the Lafayette High School, Buffalo, New York. The accompanying table summarizes the data gathered for all these groups except the Thomas Jefferson High School. What was done there is described in the quotation below from the October 1929 number of the Bulletin of High Points, in which the chairman of the mathematics department describes the investigation.

*Correlation Between Scores on the Algebra Prognosis Test and (1) Marks on the Algebra Achievement Test and (2) the Teachers' Marks, September 1928, February and September 1929 and February 1930.*

| Date | School | N | $r$ Between Prognosis Test Scores and Achievement Test Marks | $r$ Between Prog. Test Scores and Teachers' Marks |
|---|---|---|---|---|
| Sept. 1928 | Monroe | 29 | $.51 \pm .082$ | $.67 \pm .068$ |
| | | 47 | $.66 \pm .058$ | $.81 \pm .024$ |
| | | 38 | $.79 \pm .040$ | $.73 \pm .049$ |
| | Washington | 123 | $.68 \pm .036$ | $.68 \pm .036$ |
| | | 100 | $.50 \pm .051$ | $.62 \pm .041$ |
| | Bryant | 84 | $.60 \pm .048$ | $.36 \pm .065$ |
| Feb. 1929 | Washington | 67 | $.64 \pm .052$ | $.68 \pm .045$ |
| | | 54 | $.62 \pm .058$ | $.48 \pm .072$ |
| | Dickinson | 280 | $.65 \pm .023$(a) | $.59 \pm .026$(c) |
| | | | $.64 \pm .024$(b) | $.06 \pm .025$(d) |
| | | | | $.54 \pm .029$(e) |
| Sept. 1929 | Washington | 95 | $.74 \pm .031$ | $.75 \pm .031$ |
| | | 38 | $.80 \pm .040$ | $.92 \pm .017$ |
| | Buffalo | 44 | $.64 \pm .060$ | .................. |
| Feb. 1930 | Washington | 83 | $.61 \pm .048$ | $.58 \pm .050$ |
| | | 71 | $.68 \pm .044$ | $.69 \pm .042$ |
| | | 68 | $.62 \pm .050$ | $.42 \pm .067$ |
| | | 57 | $.72 \pm .043$ | $.63 \pm .053$ |

In interpreting the data in this table the following facts must be noted. The Washington and the Bryant groups used the Columbia Research Bureau Algebra Test I as the achievement test. The other schools used their own school examinations. In the Dickinson High School only the first ten parts of the prognosis test were administered because of lack of time. For this school item (a) in the table is the correlation with the midterm examination, item (b) the

correlation with the final examination, item (c) with teachers' marks at the end of the first marking period, item (d) with teachers' marks at the end of the second marking period and item (e) with teachers' marks at the end of the term.

Concerning the experiment in the Thomas Jefferson High School the chairman of the mathematics department writes as follows: "The tests were given to two groups of pupils, the first, a class of forty-four taught by the writer. These were the commercial pupils who had selected algebra as an additional study. The second group was made up of the seven classes of second term pupils in the academic course, three hundred sixty-six in all. The figures for each class were studied independently and then a summary was made for the entire group. The names were arranged according to the rank on the prognosis test, the rank in the achievement test being noted in a separate column. If the pupil ranked among the first ten in both tests, a check was placed in the next column. If he was among the first twenty in both tests, another check was added. If there was no correspondence, a cross was placed in the space instead of a check. A study of these checks and crosses shows six out of the first ten in the prognosis test also ranked among the first ten in achievement, while seventeen of the first twenty rank among the first twenty in achievement. Only one pupil in this class failed, and she was the one who was rated lowest in both tests.

In the case of the second group, out of the highest ninety-one in the prognosis test sixty-two came in the highest quarter of the achievement test scores; and of the first 182, 152 came into the upper half of the distribution of the marks on the achievement test. In no class did any of the five ranking lowest in the prognosis test come into the upper half of their class, and in only two cases did pupils who rated among the first five in the class on the prognosis test drop into the lower half in achievement, and they did not fall below the lowest quartile."

The geometry prognosis test was tried out in preliminary form in January 1928 with about four hundred students in the George Washington, the James Monroe, the New Utrecht, the Franklin K. Lane and the Thomas Jefferson High Schools, New York City. It was given during the first two recitation periods of the term before there had been any instruction in geometry. Repeaters (pupils who had failed and were taking the work a second time) were not

included. As measures of achievement scores were taken on an objective achievement test in geometry given at the end of the term. As a result of this, both the prognosis test and the achievement test were revised. Two new parts, Lesson and Test 8 and Test 10, were added and all the other parts were made longer. The prognosis test in its present form is more than twice as long as the original test and the time allowance is greater. This increases the validity of the test and has the effect of making still closer the correlation between the scores on the prognosis test and the objective measures of achievement in geometry. The achievement test was completely revised and put into its present form. The test was given again to groups of pupils in the George Washington High School in February and September 1929 and in February 1930. The accompanying tables give the data for this part of the investigation.

*Correlations Between Scores on the Geometry Prognosis Test and Marks on the Geometry Achievement Test in the Preliminary Part of the Study in Several High Schools in New York City, in Februaro 1928.*

| School | N | $r$ Between Scores on the Prognosis Test and Marks on the Geometry Achievement Test |
|---|---|---|
| George Washington | 73 | $.55 \pm .057$ |
| | 79 | $.45 \pm .060$ |
| | 47 | $.58 \pm .064$ |
| Thomas Jefferson | 59 | $.50 \pm .066$ |
| James Monroe | 44 | $.65 \pm .062$ |
| New Utrecht | 50 | $.60 \pm .061$ |
| Franklin K. Lane | 52 | $.61 \pm .060$ |

The Pearson coefficient of correlation used in evaluating the prognosis tests against scores of the criterion takes into consideration the standing of the individuals as pupils who received a particular rating at the end of the term. In a practical situation the question that arises is not, "What rating will this pupil receive at the end of the semester?, but rather, "Should this boy or girl be permitted to take up the study of algebra or of geometry, or should he be advised not to do so?" Since there is a passing mark in the schools by which the pupils are measured as successful or unsuccessful, the author evaluated the prognosis tests according to the end-term records grouped as passing or failing. For this purpose the bi-serial $r$ is used when one of the characters in a table is quantita-

*Correlations Between Scores on the Geometry Prognosis Test and Marks on the Geometry Achievement Test in the George Washington High School, New York City, in February and September 1929 and in February 1930.*

| Date | N | Between Scores on the Geometry Prognosis Test and Marks on the Achievement Test | Between Scores on the Geometry Prognosis Test and Teacher's Marks |
|---|---|---|---|
| February 1929 | 102 | $.76 \pm .029$ | $.71 \pm .034$ |
| | 66 | $.62 \pm .052$ | $.64 \pm .050$ |
| | 74 | $.61 \pm .050$ | $.62 \pm .049$ |
| | 36 | $.47 \pm .088$ | $.72 \pm .055$ |
| | 30 | $.70 \pm .063$ | $.62 \pm .076$ |
| | 66 | $.48 \pm .064$ | $.42 \pm .068$ |
| September 1929 | 41 | $.73 \pm .050$ | $.64 \pm .063$ |
| | 41 | $.48 \pm .082$ | $.61 \pm .067$ |
| | 45 | $.63 \pm .061$ | $.57 \pm .068$ |
| | 51 | $.53 \pm .069$ | $.48 \pm .073$ |
| February 1930 | 35 | $.73 \pm .053$ | $.78 \pm .045$ |
| | 44 | $.42 \pm .078$ | $.46 \pm .080$ |
| | 34 | $.29 \pm .105$ | $.64 \pm .068$ |
| | 100 | $.70 \pm .035$ | $.67 \pm .037$ |
| | 29 | $.66 \pm .070$ | $.58 \pm .082$ |
| | 117 | $.72 \pm .031$ | $.76 \pm .027$ |

tive and the other consists of two qualitative categories. The accompanying table gives the values of the bi-serial $r$ computed by means of the formula $r = M_1 - M_2/\sigma \times pq/z$ in which $M_1$ is the mean of the distribution of the passing group, $M_2$ the mean of the failing group, $\sigma$ the standard deviation of the entire distribution, $p$ the ratio of the number of individuals in the passing group to the total, $q$ the ratio of the number in the failing group to the hotel, and $z$ the ordinate of the normal probability curve which separates the two areas representing $p$ and $q$.

One of the factors that makes the bi-serial $r$ lower than it should really be is the fixed passing mark. Many a pupil with a low prognosis test score is given a mark of 65 per cent as "the benefit of a doubt" and is allowed to try the work of the next grade. The fact that such a pupil fails the next grade shows that the 65 is really a failing mark. There is no doubt in the mind of the writer, as a result of thirteen years of experience in the supervision of teachers, that this interfering factor would be removed, if the teachers were asked merely to list their pupils in the order of achievement, without paying any attention to the question of passing or failing. At present the tendency exists to include in the passing group many

*Table of Bi-serial r's for the Various Groups in the Investigation with the Algebra and Geometry Prognosis Tests in the George Washington High School, New York City, 1928, 1929 and 1930.*

| Date | Algebra | | Geometry | |
|---|---|---|---|---|
| | *N* | *r* | *N* | *r* |
| September 1928 | 102 | .79 ± .042 | — | — |
| | 120 | .81 ± .037 | — | — |
| February 1929 | 69 | .66 ± .067 | 30 | .65 ± .069 |
| | | | 102 | .73 ± .049 |
| | | | 65 | .60 ± .075 |
| | | | 74 | .62 ± .069 |
| September 1929 | 133 | .67 ± .049 | 51 | .51 ± .096 |
| | 38 | .91 ± .049 | 45 | .42 ± .122 |
| | | | 41 | .80 ± .068 |
| | | | 41 | .68 ± .092 |
| February 1930 | 82 | .73 ± .055 | 35 | .84 ± .081 |
| | 71 | .74 ± .067 | 118 | .68 ± .054 |
| | 57 | .70 ± .074 | 100 | .76 ± .061 |
| | 70 | .68 ± .034 | | |

a pupil who really deserves a mark between 50 and 60 per cent.

What is the significance of the coefficients of correlation listed in the various tables? Are they sufficiently high?[6] What do they tell about the value of using the prognosis tests for guidance in the selection of courses in school? One might apply the correction for attenuation and other statistical devices, in order to obtain the highest possible coefficients under ideal conditions. The writer, however, does not consider this a legitimate part of this study, because he is concerned with the real and not the ideal situation. The values of *r* obtained in this investigation serve as a picture of conditions as they exist in schools with large classes and with teachers of varying ability. The very fact that the coefficients vary so much for different groups during the same semester shows that the teacher element is one of the interfering factors. What Professor Percival Symonds said about the foreign language prognosis tests may also be said here:[7] "When one considers that other factors besides ability condition achievement, such as interest in subject,

[6] A discussion of the coefficient of alienation will shed light on this question. This may be found in any standard book on educational measurement.

[7] Teachers College Record, March 1930, pp. 550, 551.

school attitude, competing activities, home conditions, one is willing to make allowance for lack of perfect prediction of a mere test of ability. There is a great mortality also between the initial prognosis test and the final achievement test. This elimination affects principally those with the lower initial ability. It thus tends to reduce the variability of the groups for which the correlations have finally been computed, and hence attenuates the coefficients of correlation. Were it possible to test with the final achievement test the whole group that originally took the prognostic test, the correlations would undoubtedly be higher."

### Records of Failing Pupils

More important than the coefficient of correlation as such is the record actually made by pupils who did poorly in the prognosis tests; for, while a prognosis test is used theoretically to determine a pupil's success in a particular subject, the practical school administrator, in applying the results, should be concerned particularly with those who make the lowest scores. A pupil who can more or less master by himself in a limited number of minutes the material contained in the test ought certainly to succeed in the course of a term with the aid of a teacher. Such a pupil's failure is not due to inability. As far as the subject is concerned, it is entirely possible for him to succeed. It is the pupil at the other end of the scale who is to be guided by the test. If the child cannot grasp by himself in a few minutes a simple lesson in algebra or in geometry clearly explained, what are the chances that he will do the work successfully in the course of a term?

Past experience in the New York City High Schools has been that about thirty per cent of the pupils fail in the first term of algebra and in the first term of geometry. Allowing a margin of five per cent for the cases that might be doubtful, the writer studied the records of the lowest quarter in each of several groups. In the algebra group there were 194 pupils in the lowest quarter of the distribution of the prognosis test scores. Of these, 167 failed at the end of the term, 18 received the bare passing mark and 9 received a mark that was five points higher than the passing mark. In the geometry group, out of 223 pupils in the lowest quarter of the distribution of the prognosis test scores, 156 failed at the end of the term, 34 received the bare passing mark and 33 received a mark that was five or ten points higher than the passing mark.

These figures seem to the writer to be very significant. When one considers that the teachers did not know the prognosis test scores throughout the term, the predominance of failing ratings indicates that the poor work done by these pupils was foretold by the test itself. In the algebra group 96 per cent of teacher's marks for pupils in the lowest quarter of the distribution are failing, ten per cent are just passing and four per cent are above the passing mark. In the geometry groups 70 per cent of the marks are failing, fifteen per cent are just passing and fifteen per cent are above the passing mark. This latter fifteen per cent may be explained by the fact that geometry requires greater reaction time than algebra. It is quite possible, therefore, that some pupils may not do their best on the prognosis test, who in the course of a term find a better opportunity to grasp the subject. There is no doubt that in time the test may be improved and such a situation remedied.

Since the bare passing mark may be looked upon with suspicion as far as successful achievement is concerned, especially when an achievement test rating or the record of the following term do not justify the passing mark, we may say that in algebra almost 95 per cent and in geometry 85 per cent of the pupils did work that agreed with the prognosis test results, in so far as their standing in the lowest quarter of the distribution was accompanied by failure at the end of the term.

It is significant also to know what per cent of all the failures in each group was in the lowest quarter of the distribution of scores on the prognosis test. In the algebra group of a total of 299 failures at the end of the term, 167 or 56 per cent were in the lowest quarter of the distribution of the prognosis test scores; in the geometry of a total of 274 failures, 157 or 57 per cent were in the lowest quarter.

If a test of specific ability is used for locating the probable failures and guiding them accordingly, then the more failures it locates, the better it serves its purpose. Even a perfect instrument will not locate all the failures, since some of the pupils with high scores on the prognosis test will not succeed in the course of a term because of the factors in the teaching situation which cannot be controlled. The fact that close to 60 per cent of all the failures were found in the lowest quarter of the distribution of both the algebra and the geometry test scores indicates the possibility with a test of specific ability.

### The Use of Intelligence Tests

Since attempts have also been made to predict success in algebra and in geometry on the basis of intelligence tests and of achievement in arithmetic, the following data are included for purposes of comparison. The correlations obtained in connection with the use of the Otis Self-Administering Test were as follows:

| | |
|---|---|
| a) between I.Q. and algebra prognosis test | $.58 \pm .030$ |
| b) between I.Q. and teacher's marks in algebra | $.54 \pm .032$ |
| c) between I.Q. and geometry prognosis test | $.64 \pm .030$ |
| d) between I.Q. and geometry prognosis test | $.62 \pm .030$ |
| e) between I.Q. and teacher's marks in geometry | $.51 \pm .050$ |
| f) between I.Q. and teacher's marks in geometry | $.54 \pm .046$ |

While these correlations do give one a picture of the relationship between intelligence and achievement in algebra and in geometry, it seems to the writer that the following figures are more meaningful. In September 1930, after the prognosis test had been administered, the lowest twenty per cent of each group and also the highest twenty per cent of the algebra group were segregated into separate classes for purposes of instruction. The range of I.Q.'s in the slow algebra class was 71 to 123, in the slow geometry class 81 to 120 and in the rapid algebra class 95 to 131. A comparison of these figures with those representing an entire group in each of algebra and geometry shows that slow classes formed on the basis of specific ability take in a wide range of I.Q.'s. Since the prediction of success for the purpose of guidance in the election of studies concerns the individual pupils, the method of segregation should be used which will be most beneficial to the individual.

### The Use of Arithmetic Ratings

The question is constantly asked as to why prognosis of success in algebra cannot be based upon the child's preparation in arithmetic. Reference was made to this problem in the early part of this report. Evidently there is something more involved in the study of algebra than the mere mastery of whatever is taught in arithmetic. In this study the writer assumed that the pupil's elementary school rating in arithmetic at the end of the eighth year is an index of his preparation in the subject. In the New York City schools each child is given a double rating in arithmetic, e.g. A A or A B or C B etc., one for mechanical computation and the other for

problems. These ratings were changed into numerical equivalents by means of a value of 5 for an A, 4 for a B+, 3 for a B, 2 for a C and 1 for a D. A mark of A A was, therefore, equivalent to 10, B B+ to 7, etc. The writer then computed the coefficient of correlation between these numerical equivalents and the teachers' marks in algebra at the end of the term and found them to .49, .48, .48, .33 for four groups with 211, 54, 76 and 247 cases respectively.

It seemed quite possible that a different assignment of values to the literal ratings in arithmetic might have produced other coefficients of correlation. And so, the distribution was made of the percentage of each combination of literal ratings that appeared in the entire group that had 247 cases and the $x/\sigma$ values were computed from the table of areas of the normal probability curve in terms of deviates from the mean. The correlation between the $x/\sigma$ scores and the algebra marks at the end of the term was found again to be .33.

Treating the I.Q.'s and the arithmetic marks just as the prognosis test scores were treated, the writer also investigated the number of failing pupils who were in the lowest quarter of these distributions. For the distribution of I.Q.'s, 43 per cent of the total number of failures in algebra were in the lowest quarter, and 49 per cent of the total number of failures in geometry. For the distribution of the elementary school arithmetic marks, 36 per cent of the total number of failures in algebra were in the lowest quarter.

It is well to compare the percentage of failure found in the lowest quarter of each of the three distributions—of the test of specific ability, of the intelligence test and of the elementary school arithmetic ratings. While the coefficients of correlation do tell something about the groups involved in the investigation, the important thing, in a study of this sort, is to see what prognosis can be made particularly for the weaker pupils who should not be permitted to enter upon the study of the subject. The instrument which will enable the schoolman to locate the probable failures with greater certainty is the instrument he should use. It seems to the author significant, therefore, to see that the test of specific ability puts into the lowest quarter of its distribution almost 60 per cent of the failures both in algebra and in geometry, as compared with 43 and 49 per cent respectively for the distribution of I.Q.'s and 36 per cent for the distribution of the elementary school arithmetic marks.

The difference between these various figures seems to favor the test of specific ability.

Since intelligence is a factor in the study of mathematics, it is well to see what value there is in a combination of the prognosis test and the intelligence test and possibly also the arithmetic marks for the purpose of prediction. Ferdinand Kertes of the Perth Amboy, N. J. High School, with the same test of specific ability as used by the writer, found the following correlations:

a) between I.Q. and algebra marks (219 cases) . . . . . . . . . . . . . . . . . $.56 \pm .037$
b) between prognosis test and algebra marks (114 cases) . . . . . . . . . $.61 \pm .040$
c) between I.Q. and algebra marks (114 cases) . . . . . . . . . . . . . . . . . $.50 \pm .047$
d) between algebra marks and a combination of I.Q. and arithmetic marks (112 cases) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . $.66 \pm .038$
e) between algebra marks and a combination of I.Q. and prognosis test (112 cases) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . $.68 \pm .034$
f) between algebra marks and a combination of prognosis test and arithmetic marks (112 cases) . . . . . . . . . . . . . . . . . . . . . . . . . . $.72 \pm .031$
g) between algebra marks and a combination of prognosis test, I.Q. and arithmetic marks (112 cases) . . . . . . . . . . . . . . . . . . . . $.68 \pm .034$

The difference between .61 and .72 would seem to indicate that the combination of the prognosis test and the arithmetic marks would be a better basis for prediction than the specific ability test alone. This particular study seems to show that the addition of the intelligence test is of little importance.

In a somewhat similar study made by C. C. Grover in Oakland, California, he found the correlation between the scores on the prognosis test in algebra and on the achievement test to be .61 and between the scores on the achievement test and a combination of the prognosis test and the Terman Group Test to be .65. This points to the combination of the two as slightly better for the purpose of prognosis than the specific ability test alone.

The writer himself, in the case of a group of 213 pupils in geometry, found the correlation between scores on the geometry prognosis test and on the achievement test to be .71, and between the latter and a combination of the prognosis test and the Otis Intelligence Test to be .72.

Since the writer feels that the importance in the use of any test for the purpose of prognosis in mathematics from a practical standpoint is the distinction between passing and failing, he has not emphasized at all the matter of the prediction of scores. He is not interested in knowing whether a pupil who will probably succeed is

going to attain a score of 75 or 78 etc.; and if the test indicates that a certain pupil is going to fail, of what importance is it to know his probable score? However, a study of this phase of the question was made by C. C. Grover referred to above. In his report (unpublished) he writes, "The regression equation for predicting the achievement test score of a particular pupil from his prognostic test score is $\overline{X}_1 = .11\overline{X}_2 + 9$ where $\overline{X}_1$ is the predicted achievement test score, $\overline{X}_2$ the score on the prognostic test and 9 a constant. The probable error of estimate of this predicted score is 2.4. As an example of the use of this regression equation, the records of several students are given here:

| Score on Algebra Prognosis Test | Estimated Score on Achievement Test | Actual Score on Achievement Test |
|---|---|---|
| 125 | $\overline{X}_1 = (125 \times .11) + 9 = 23$ | 22 |
| 94 | $\overline{X}_1 = (\ 94 \times .11) + 9 = 19$ | 17 |
| 83 | $\overline{X}_1 = (\ 83 \times .11) + 9 = 18$ | 15 |

The above cases are merely illustrative of the use of the equation, and it is not to be expected that the correspondence would be as close in unselected cases."The regression equation for predicting the achievement scores in terms of intelligence and prognostic test scores is $\overline{X}_1 = .09\overline{X}_2 + .07\overline{X}_3 - 7.2$, where $\overline{X}_1$ is the predicted achievement test score, $\overline{X}_2$ the score on the prognostic test, $\overline{X}_3$ the intelligence quotient, and 7.2 a constant. The probable error of the estimated score is 2.36. The use of this equation is illustrated as follows:

| I.Q. | Prognosis Test Score | Estimated Achievement Test Score | Obtained Ach. Test Score |
|---|---|---|---|
| 135 | 165 | $\overline{X}_1 = (.09 \times 165) + (.07 \times 135) - 7.2 = 17$ | 17 |
| 134 | 131 | $\overline{X}_1 = (.09 \times 131) + (.07 \times 134) - 7.2 = 14$ | 15 |
| 129 | 149 | $\overline{X}_1 = (.09 \times 149) + (.07 \times 129) - 7.2 = 15$ | 14 |

As was the case above, an unselected group would show a much wider divergence between predicted and obtained scores, and those chosen are merely illustrative of the results to be expected under the most favorable conditions."

## Individual Records

The writer feels that a study of the records of individual pupils over a number of terms may also be significant. The pupils chosen are the ones who stood at the lower end of the lists in the prognosis tests. Each mark in the following list represents one semester's work. The passing mark is 65. Any mark below 65 represents failure. Each line represents one pupil.

| Algebra | | Geometry | |
|---|---|---|---|
| First Half | Second Half | First Half | Second Half |
| 35, 75 | 65 | 40, 50, 65 | Dropped |
| 65 | 65 | 55, 65 | 70 |
| 50, 65 | 65 | 40, 70 | 65 |
| 65 | 65 | 40, 85 | 50 |
| 65 | 65 | 40, 65 | 65 |
| 40, 40 | Dropped | 40, 70 | 65 |
| 65 | 55, 70 | 65 | 50 |
| 30, 55 | Dropped | 40, 70 | 40 |
| 50, 65 | 65 | 20, 65 | 35 |
| 70 | 55, 55 | 50, 65 | 70 |
| 40, 55 | Dropped | 50, 65 | 50, 75 |
| 50, 65 | 30 | 30, 70 | Dropped |
| 40, 65 | 65 | 50, 65 | 55, 65 |
| 65 | 55, 70 | 70 | 55, 75 |
| 45, 80 | 55 | 40, dropped | |
| 45, 70 | 65 | 50, 55 | — |
| 45, 85 | 65 | 40, 50, 65 | Dropped |
| 55, 75 | 65 | 55, 65 | 70 |
| 50, 55, 65 | 50 | 40, 70 | 65 |
| 30, 85 | 50, 65 | 40, 65 | 65 |
| 40, 65 | 50 | 40, 70 | 40 |
| 45, 65 | 70 | 20, 65 | 30 |
| 20, 30, dropped | | 55, 65 | 70 |
| 20, 55, dropped | | 30, 75 | 65 |
| 50, 70 | 55 | 40, 65 | 55, 65 |
| 50, 80 | 65 | 50, 65 | 65 |

This list is only representative of a much larger group. What a saving of time and of energy could have been brought about for these children, if they had been guided early enough into something they might have been able to do better.

## Slow Classes

The practical administrator will ask, "Now that I have located the pupils who will probably not succeed in the study of the usual course in algebra or in geometry, what shall I do with them? Shall

they be dropped from the classes entirely, or shall they be grouped separately and be given a modified course in the subject matter?" In attempting to find an answer to this question, the writer formed a slow group in algebra and one in geometry in February 1930. The teachers of the geometry classes followed an outline based upon the regular course, but which included only such topics as were considered essential for a very minimum course in geometry. In the algebra class the teacher used a recent textbook in which the explanatory material served as a reading lesson for the pupils and they practically studied their work slowly in this way under the direct supervision of the teacher. At the end of ten weeks the pupils wrote an examination based upon the work covered in the class. The same examination was given to an equal number of pupils in a normal group. Although this examination was based upon work that was easier than that done in the normal classes, some of the pupils in the latter made low scores. This was due to the fact that, because of program exigences, pupils who did not take the prognosis test remained in the normal groups. Some of these no doubt should have been assigned to the slow classes.

The most significant thing about the distribution of scores in this examination is the number of boys and girls who were failing even in this modified diluted course. If this is typical of what one would find generally, then a certain percentage of the pupils ought to be eliminated from the mathematics courses altogether. As far as the others in the group were concerned who were doing satisfactory work, one may well picture their position as failures at the lower end of the list in a normal group. If, therefore, it is to be the policy of a school to give mathematics to all who can do it with success, this study would seem to indicate that a certain percentage of the pupils should be classified as a slow group and be given work according to their ability. A graph of the results of the examination would show how much better the normal groups responded to the examination, even though the questions covered only a small part of the work done during the ten weeks. The slow group was especially prepared for the test, and still far short of the normal group. In the geometry thirty per cent of the normal group were better than the best of the slow group and fifty per cent were better than ninety per cent of the slow group. In the algebra thirty-seven per cent of the normal group were better than the best of the slow group and sixty-two per cent of the normal group were bet-

ter than eighty-five per cent of the slow group. This seems to point to classification as the better provision for the slow pupils.

In connection with a discussion of the classification, segregation and elimination of pupils on the basis of a test of specific ability, it is well to consider whether it is worth while for the teacher to know the pupils' scores on the prognosis test, even though the class remains a heterogeneous group. Throughout this investigation all the groups were of this sort, except the slow and the rapid groups referred to. During the term beginning September 1930 the teachers were acquainted from the start with the standing of the pupils on the prognosis tests. The correlations between the scores and the teachers' marks at the end of ten weeks were all between .65 and .70. It is difficult to conclude from these anything definite about the value of knowing the scores in advance. Since these correlations, however, are higher than many that appear in this study, one may be led to feel that the difference is due to the fact that the teachers were in a position to follow the able pupils who were not working as hard as they could. Many a pupil who ranks high in the prognosis test, although considered passing at the end of the term, does not maintain the same relative position on the list of the teacher's marks or of the achievement test scores. This may be due mainly to the fact that he has not worked in accordance with his ability. Such cases in a group tend to lower the coefficient of correlation between the prognosis test scores and the criterion. In the groups in which the teachers knew the prognosis test scores from the beginning of the term, they had a check on the pupil who stood high on the prognosis test, but who was not doing well in class. If he was able to concentrate and to acquire by himself in a limited number of minutes enough of each lesson in the test to obtain a good score, then he should do satisfactory work throughout the term. It is quite possible that through further effort on the part of the teacher during the second half of the term, the coefficients of correlation obtained for the first ten weeks would grow appreciably higher. The increased effort in behalf of the better pupils may be balanced by a corresponding saving of energy with the very weak ones. If these pupils do unsatisfactory work, the teacher may quite conscientiously devote to them a minimum of time and of energy, just enough to develop their capabilities and to keep them from interfering with the work of the rest of the class.

## Practical Value of the Study

The outcomes of this investigation are the following:

1. The correlations between scores on a test of specific ability in algebra or in geometry and marks representing achievement are in general higher than those between I.Q.'s and marks in achievement.

2. The correlations between scores in the algebra prognosis test and marks of achievement are much higher than those between elementary school arithmetic marks and marks of achievement in algebra.

3. A study of the location of failing pupils in the lowest quarter of the distribution of the three instruments studied indicates a preference for the test of specific ability, with the intelligence test second and the arithmetic marks third.

4. The combination of the prognosis test and the intelligence test serves as a slightly better basis for prediction in geometry than the former alone. It is a question as to whether the slight difference will appeal to the practical administrator who is interested in conserving time and expense.

5. The combination of the prognosis test and the arithmetic marks serves as a better basis for prediction in algebra than the former alone.

6. The range of I.Q.'s of a group of poor pupils segregated on the basis of the test of specific ability is wide enough to make one feel that segregation on the basis of the I.Q.'s would do a grave injustice to a number of pupils.

7. Even in a group of poor students who are given a modified course at a much slower pace than is ordinarily followed, there are some who are not successful. This would seem to point to the need for eliminating the very poor ones at the beginning of the term. The test of specific ability seems to predict the failure of these pupils.

8. The knowledge of the prognosis test scores by the teachers from the very start seems to be helpful in the heterogeneous groups, since they seem to guide the teacher in the treatment of the pupil.

9. A study of the records of individual pupils at the lower end of the distribution of the prognosis test scores over a period of

several terms indicates the value of a test of specific ability for guidance.

What Bobbitt said in 1918 certainly applies today. "We must recognize the fact that individuals differ in their natural capacities. No amount of educational labor will develop large ability on the part of those possessing low natural capacity. For these we shall be compelled to determine a limited set of abilities and we shall have to aim at only a moderate, or even, low standard of achievement in those abilities. Those with large potential capacity should have their powers fully unfolded."[8]

[8] Bobbitt, J. F. The Curriculum. Houghton Mifflin Co., 1918.

## NOTICE TO SUBSCRIBERS

This is the last issue of *The Mathematics Teacher* for the academic year 1933-34. If your subscription expires with the May number please renew it at once so as not to miss the October and following numbers. The month in which your subscription expires is printed on the outside wrapper of every issue of the *Teacher*. Will you also give us your change of address if you have moved recently so that you will not miss any future numbers.

Subscribers who will be attending meetings of mathematics teachers this coming fall should send for advertising circulars to the office of *The Mathematics Teacher*, 525 West 120th Street, New York City.

We should like to thank the numerous subscribers who have sent in for circulars during the past year for their interest in the work of The National Council.

# The 15th Annual Meeting of the National Council of Teachers of Mathematics, Cleveland, Ohio, February 23-24, 1934

EDWIN W. SCHREIBER, *Secretary*

*Directors' Meeting—February 23, 10:00 a.m.*
*Hotel Cleveland, Conference Room 24*

PRESENT: William Betz, Mary A. Potter, Ralph Beatley, Edwin W. Schreiber, Vera Sanford, Mary S. Sabin, C. Louis Thiele, John P. Everett, Elsie Parker Johnson, Raleigh Schorling, Harry C. Barber, J. O. Hassler, W. S. Schlauch.

ABSENT: W. D. Reeve, Herbert E. Slaught, Marie Gugle.

President William Betz, of Rochester, New York, called the meeting to order shortly after ten o'clock. The first item of business was the reading of the minutes of the 14th annual meeting, by Secretary Schreiber, which was held in Minneapolis, Minnesota, February 24, 25, 1933. The minutes were approved as read. The treasurer's report was presented, discussed, and approved by the Board of Directors. The complete report is to be found at the end of these minutes. The auditor, Dr. Vera Sanford, reported that she had carefully examined the accounts of the Treasurer and found them correct. Professor Schlauch suggested that in the future the book value of the bonds held by the National Council should be apportioned according to regular amortization schedule. This suggestion was later put in the form of a resolution and was carried. The matter of year books was thoroughly discussed and the following resolution proposed by Professor Schorling was approved: Moved that it is the conviction of the Board of Directors that the materials included in the various year books are necessary to an adequate training for prospective teachers of mathematics, and that the president and secretary be instructed to communicate with instructors of method classes in mathematics in teacher training institutions, urging them to supply their institutions with a set of year books. Furthermore that the president, secretary, and editor-in-chief be authorized to fix the price of sets of year books, and that they make the same attractive to teacher training institutions but at the same time, making sure that the price includes all costs of publishing and handling. A motion was made and carried to compile a mailing list of all instructors of teacher training classes in mathematics in our colleges and universities, and eventually circularize these institutions with information concerning the year books of the National Council. Owing to the fact of extreme inclement weather in New York City, the ninth year book was not available in time to be distributed at the 15th annual meeting. The chairman of the Ballot Committee, Secretary Schreiber, gave a brief report and stated that he thought we had too much machinery for our annual ballot which for this year consisted of 174 votes. After some discussion, the Board of Directors instructed the Ballot Committee to proceed in their functions as in the past. Professor Schorling made a brief report for the Committee on Individual Differences, stating that some good work had been done and with no expense to the National Council. On motion

the committee was continued. A resolution was passed to the effect that all standing committees continue to function until they are discharged. Professor J. O. Hassler made a brief report for the Policy Committee, stating that the committee was now thoroughly organized and was ready to make a report on Saturday afternoon. President Betz gave a brief report on the summer meeting held in Chicago, July 3, 1933, in connection with the N.E.A., and the joint dinner with the Central Association of Science and Mathematics Teachers. This was one of the best attended meetings the National Council has ever held. A report of this meeting is to be found in the News Notes of THE MATHEMATICS TEACHER, November, 1933. President Betz also reported on the Boston meeting held in December, 1933 at which time the National Council was invited to a round table discussion by the Mathematical Association of America. The matter of the time of the annual meeting was discussed and there seemed to be some division in the Board as to meeting with the A.A.A.S. in December or the N.E.A. in February. The president read a cordial invitation he had received from Pittsburgh to the effect that we meet in Pittsburgh next December. It was finally voted that an appropriation of $150 be set aside for the Pittsburgh meeting. Mr. Everett proposed several suggestions with reference to the office of president and secretary but no official action was taken. The meeting then adjourned. The Directors' Luncheon was held at 12:30 in the Empire room of the Cleveland Hotel.

*First General Meeting—February 23, 2:00 p.m.*
*Hotel Cleveland, Empire Room*

President Betz presented Professor Ralph Beatley of the Graduate School of Education, Harvard University, and chairman of the Geometry Committee, who made the report of his committee on "The Future of Geometry in the High School." Professor Beatley presided for the balance of the afternoon and introduced as the second speaker, Mr. Rolland R. Smith, head of the Department of Mathematics, Central High School, Springfield, Massachusetts. Approximately 125 persons were in attendance at this session. Considerable discussion from the floor followed these reports.

*Dinner for Official Delegates*
*in charge of*
*Mrs. Elsie Parker Johnson, Oak Park, Illinois*

This annual affair for the official delegates and members of the Board of Directors has proved its value to the Council because of the opportunity for reports from delegates from local organizations. Mrs. Johnson called upon the following who made brief reports: S. Helen Taylor, C. J. Leonard, Foreman W. Slayer, A. J. Griffith, Alfhild Alenius, William Betz, Clara D. Murphy, Edith Woolsey, J. O. Hassler, A. Brown Miller, Ruth Lane, H. Carlisle Taylor, Marie Becker, D. Talmage Petty, C. M. Austin, Nanette Roche, H. P. Fawcett, Fred Bedford, and Sara C. Walsh. The following were also in attendance at the dinner: Vera Sanford, Norma Struke, Raleigh Schorling, Edna Vokes, W. S. Schlauch, John P. Everett, Mrs. F. Brooks Miller, H. C. Barber, C. L. Thiele, Mrs. Fred Bedford, Dora E. Kearney, Martha Hildebrandt, Agnes Herbert, Arthur S. Otis, Edwin W. Schreiber, Elsie Parker Johnson, J. T. Johnson, Mary A. Potter, Mary W. Crofts, Mary S. Sabin, Ethel Harris Grubbs, H. E. Benz, Alma Wuest, and Theresa L. Podmele.

*Second General Meeting—February 23, 8:00 p.m.*
*Hotel Cleveland, Empire Room*

Greetings on Behalf of Reception Committee— Mrs. Florence Brooks Miller, Chairman Reception Committee.

Address of Welcome—Mr. Charles Lake, Superintendent of Schools, Cleveland.

Response on behalf of the National Council—Professor Ralph Beatley, Second Vice-President National Council.

"Panel" Discussion on the General Theme—The Present Crisis in Secondary Mathematics, directed by Miss Mary A. Potter, First Vice-President, and assisted by the following "jury":

Mr. Harry C. Barber, Dr. John P. Everett, Miss Mary S. Sabin, Prof. W. S. Schlauch, Miss Martha Hildebrandt, Prof. Raleigh Schorling, Mrs. Elsie Parker Johnson, Prof. Edwin W. Schreiber, Mr. C. Louis Thiele, and Dr. Vera Sanford

*Annual Business Meeting, February 24, 9:00 a.m., Red Room*

President Betz called the meeting to order and instructed the secretary to read the minutes of the last annual meeting. A motion from the floor was made to the effect that the minutes be approved as printed in the November issue of THE MATHEMATICS TEACHER. The motion was carried. Treasurer Schreiber then made the annual financial report of the National Council which was accepted as read. The Chairman of the Ballot Committee, Secretary Schreiber, presented the report of this committee with the following results: out of 174 votes cast—

FOR PRESIDENT

| *Votes* | | *Votes* | |
|---|---|---|---|
| 97 | Hassler, J. O., Norman, Okla. | 71 | Sanford, Vera, Oneonta, N. Y. |

FOR SECOND VICE-PRESIDENT

| | | | |
|---|---|---|---|
| 83 | Congdon, Allan R.. Lincoln, Nebr. | 71 | Stokes, C. M., Philadelphia, Pa. |

FOR MEMBERS OF THE BOARD OF DIRECTORS

| | | | |
|---|---|---|---|
| 131 | Betz, William, Rochester, N. Y. | 52 | Kearney, Dora E., Cedar Falls, Ia. |
| 65 | Charlesworth, H. W., Denver, Colo. | 68 | Thiele, C. Louis, Detroit, Mich. |
| 84 | Christofferson, H. C., Oxford, O. | 73 | Woolsey, Edith, Minneapolis, Minn |

Secretary Schreiber then declared the following duly elected:

President—J. O. Hassler

Second Vice President—Allan R. Congdon

Members of the Board of Directors—William Betz, H. C. Christofferson, Edith Woolsey.

Professor W. D. Reeve presented his report as editor of THE MATHEMATICS TEACHER and the 9th Year Book which report was accepted. President Betz reported to the members some of the deliberations of the Board of Directors at their Friday morning session. The meeting then adjourned.

*Third General Meeting*
*Saturday Morning, February 24*
*Ten O'clock—Red Room*

CENTRAL TOPIC—The Problems of Ability Grouping and Differentiated Curricula.

1. Some Aspects of the Administrative Phase of this Problem.
   Dr. C. N. Stokes, Temple University, Philadelphia.
2. Remedial Work in Arithmetic, A Challenge and A Warning Signal.
   Miss Genevieve Skehan, Whitney, School No. 17, Rochester, N.Y.
3. The Problem of Individual Differences and a Suggested Attempt for its Solution.
   Miss Clara D. Murphy, Evanston Township High School, Evanston, Illinois.
4. An Experiment in Teaching Graphs (An illustrated demonstration).
   Mrs. Wilimina E. Pitcher, Rawlings Junior High School, Cleveland, Ohio.

*Fourth General Meeting*
*Saturday Afternoon, February 24*
*Two O'clock—Red Room*

Central Topic—What Can we do to Meet the Challenge of the Present Situation in Secondary Mathematics?

1. Professor William D. Reeve, Teachers College, Columbia University.
2. Discussion. A group of four-minute addresses by the following speakers: Miss Alfhild Alenius, Denver; Mr. R. P. Wray, Pittsburgh; Mr. D. W. Werremeyer, Cleveland; Mr. C. J. Leonard, Detroit; Miss Edith Woolsey, Minneapolis; Dr. Allan R. Congdon, Lincoln; Miss Ruth Lane, Iowa City; Professor G. H. Jamison, Kirksville, Mo.
3. Professor J. O. Hassler, Chairman of Policy Committee, University of Oklahoma, Norman, Oklahoma.

*Four O'clock Meeting of Board of Directors*

Present: William Betz, Ralph Beatley, Mary A. Potter, Edwin W. Schreiber, William D. Reeve, Vera Sanford, Mary S. Sabin, John P. Everett, Elsie Parker Johnson, Raleigh Schorling, Harry C. Barber, J. O. Hassler, W. S. Schlauch, Allan R. Congdon, H. C. Christofferson, Edith Woolsey, C. Louis Thiele.

Absent: Herebert E. Slaught, Marie Gugle.

President Betz called this meeting of the Board of Directors to order and the first item of business was to establish a chairman of our finance committee. It was moved and carried that the President be chairman of the Finance Committee. For this year, the Finance Committee is composed of the following:

J. O. Hassler, Chairman; William D. Reeve, William Betz, Vera Sanford, Edwin W. Schreiber.

After considerable discussion it was moved and carried that the Finance Committee be impowered to spend as much as $300 for the Pittsburgh meeting of the National Council to be held next December. It was moved and carried that Miss Martha Hildebrandt be appointed to fill the unexpired term on the Board of Directors of J. O. Hassler, who was promoted to President of the Council. The term is for two years. It was moved and carried that Professor Beatley, Chairman of the Geometry Committee, be impowered to seek such statistical advice and service for his Committee as seemed necessary with no expense to the Council. It was moved and carried that we, as the Board of Directors and the official body of the National Council extend a sincere vote of thanks to Professor William D. Reeve for his conscientious and enthusiastic work as Editor of The Mathematics Teacher and the Year

Books. It was moved and carried that the five-year contract between the National Council and William D. Reeve be continued for another five years. It was moved and carried that any adjustment necessary in the appropriation for the Secretary-Treasure office be referred to the Finance Committee with power to act. J. O. Hassler reported that Harry C. Barber is now Chairman of the Policy Committee. The meeting adjourned.

*Saturday Evening, February 24*
*Annual Banquet and Final Meeting*
*Six O'clock—Ball Room*

The annual banquet of the National Council was attended by 160 members and guests. During the course of the banquet, music was furnished by an ensemble from the orchestra of the John Adams High School, Cleveland, Ohio. At the speakers' table, each guest was given a favor, consisting of a set of Napier's Rods made by the pupils in the Junior High School at Shaker Heights, Ohio, under the direction of Mrs. Florence Brooks Miller. President Betz at the conclusion of the banquet, introduced to the audience, members of the Local Committee, honored guests, members of the Official Board whose terms of office have terminated, and past presidents of the National Council. Secretary Schreiber made a brief report as to the attendance at this 15th annual meeting, stating that 170 had registered from 20 different states. He also asked those to rise who had attended the first annual meeting in Cleveland in 1920 and the other Cleveland meetings in 1923 and 1929. Members and guests were introduced geographically also. President Betz then presented the speaker of the evening, Professor Carl A. Garabedian, St. Stephens College, Annandale-on-Hudson, New York, who gave a very stimulating address on Mathematics and Music. Dr. John P. Everett, representing the Board of Directors, presented President Betz a token of our esteem for his services as President for the past two years. Mr. Betz responded in a gracious manner and turned over the gavel and the president's badge to our new president, J. O. Hassler of the University of Oklahoma, who made a short address at the termination of which the 15th annual meeting was declared adjourned.

### Attendance

Note: See Key to Code at end of list.

*Colorado* (2)
Denver
Alenius, Alfhild (W) South High
Sabin, Mary S. (B-7) East High
*District of Columbia* (2)
Washington
Grubbs, Ethel Harris (R-6) High Schools, Divisions 10–13
Stinson, Wm. B. (W) Terrell Jr. H.S.
*Georgia* (1)
Atlanta
Carlton Frank (W) Commercial H.S.

*Illinois* (12)
Chicago
John, Lenore (W) University H.S.
Johnson, J. T. (B-8) Normal College
Petty, D. Talmage (W) F. W. Parker Sch.
Wood, Harry H. (R-5) Prairie Ave.
Evanston
Moulton, E. J. (G) Northwestern Univ.
Murphy, Clara D. (W) Township H.S.
La Salle
Short, Vivian (R-2) H.S. & Jr. Col.

Macomb
Schreiber, Edwin W. (B-11) State Teachers College
Maywood
Hildebrandt, Martha (B-7) Proviso Twp. H.S
Oak Park
Austin, C. M. (B-14) Twp. H.S.
Johnson, Elsie Parker (B-8) Twp. H.S.
Urbana
Taylor, S. Helen (W) University H.S.
*Indiana* (7)
Bloomington
Williams, Kenneth P. (G) Indiana Univ.
Fort Wayne
Fielder, Adelaide L. (W) South Side H.S.
Flint, A. V. (W) South Side H.S.
Muncie
Whitecraft, L H. (R-5) Ball State Tea. Col.
Terre Haute
Kennedy, Kathryn (B-8) Training H.S.
Morris, Inez (R-3) Ind. State Tea. Col.
Shriner, Walter O. (R-5) Ind. St. Tea. Col.
*Iowa* (2)
Cedar Falls
Kearney, Dora E. (R-6) Teachers' Col.
Iowa City
Lane, Ruth (R-2) State Univ. of Iowa
*Maryland* (2)
Baltimore
Herbert, Agnes (R-4) Clifton Park Jr. H.S.
Roche, Nanette (R-2) Supervisor of Math.
*Massachusetts* (4)
Boston
Barber, Harry C. (B-7) English H.S.
Stevnes, Ernest N. (R-4) Editor, Ginn & Co.
Cambridge
Beatley, Ralph (R-5) Harvard Univ.
Springfield
Smith, Rolland R. (R-2) Central H.S.
*Michigan* (13)
Adrian
Sister Mary Alphonsus (W) St. Joseph Coll. & Aca.
Sister Mary Francis (W) St. Joseph Academy
Sister Rose Ethel (W) St. Joseph Academy
Ann Arbor
Schorling, Raleigh (B-14) Univ. of Mich.
Birmingham
Margard, Lila S. (R-2) Bir. Jr. H.S.
Detroit
Leonard, Clarence J. (R-2) Southeastern H.S.
McNally, J. V. (R-2) MacKenzie H.S.
Thiele, C. Louis (B-8) Dept. of Supv.
Vokes, Edna (R-3) Miller H.S.
Flint
Loss, Nellie (R-3) Flint Schools
Kalamazoo
Everett, John P. (B-11) Teachers' Col.
Ypsilanti
Schnell, Leroy H. (R-3) Roosevelt
Turner, Mabel E. (R-3) State Norm. Col.
*Minnesota* (1)
Minneapolis
Woolsey, Edith (R-2) Sanford Jr. H.S.
*Missouri* (3)
Columbia
Butler, C. H. (R-2) Univ. H.S.
Kirksville
Jamison, G. H. (R-5) State Tea. Col.
Pemberton, W. S. (R-4) State Tea. Col.
*Nebraska* (1)
Lincoln
Congdon, Allen R. (R-3) Univ. of Nebr.
*New York* (17)
Annandale-on-Hudson
Garabedian, C. A. (G) St. Stephen's Col.
Buffalo
Crofts, Mary E. (R-5) Supervision
Podmele, Theresa L. (R-2) East High
Walsh, Sara C. (R-4) East High

Foustville
Wilson, Beatrice (W) High School
Geneseo
Countryman, R. L. (W) Normal School
New York City
Reeve, W. D. (B-13) Tea. Col. Columbia
Schlauch, Wm. S. (R-5) New York Univ.
Oneonta
Sanford, Vera (B-8) Normal School
Randolph
Hinsdale, Mary E. (W) Central
Rochester
Betz, William (B-11) Public Schools
Skehan, Genevieve (G) Whitney
Taylor, H. Carlisle (R-2) B. Franklin H.S.
Syracuse
Carroll, J. S. (W) Syracuse Univ.
Tarrytown
Bedford, Fred (R-3) Wash. Irving H.S.
Bedford, Mrs. Fred (G)
Yonkers
Otis, Arthur S. (R-6) World Book Co.
*Ohio* (89)
Akron
Bartlett, Frank E. (G) Old Trail
Gerber, A. J. (R-2) Buchtel H.S.
James, Henry (W) East High
Ashtabula
Stephens, Robert C. (W) City High
Athens
Benz, H. E. (R-6) Ohio Univ.
Morton, R. L. (B-7) Ohio Univ.
Pickett, Hale (R-4) Athens High
Bellevue
Bates, Florence (W) High School
Berea
Maitland, Wm. B. (G) Berea High
Bowling Green
Overman, J. R. (B-8) State Tea. Col.
Canton
Richey, C. L. (R-2) Lincoln Jr. H.S.
Cincinnati
Becker, Marie (R-5) Walnut Hills
Struke, Norma (R-5) Rothenberg Jr.
Wuest, Alma (R-2) Walnut Hills
Cleveland (53)
Baldwin, Helen E. (G) John Hay
Bletcher, Eliz. (W) Al. Hamilton Jr.
Boyce, Moffatt G. (G) West. Reserve
Boyd, Mrs. Thelma S. (G)
Brown, O. E. (G) Case Sch. A.S.
Burroughs, Fred N. (R-3) John Adams
Clark, Helen L. (G) Al. Hamilton Jr.
Coverdale, Addie (G) Willson Jr.
Coyner, Clara E. (R-2) Lincoln H.S.
Dauber, Wilma (W) Nathan Hale Jr.
Dunstan, Laura E. (W) Substitute
Fowler, Katharine (R-2) John Adams
Gaylord, Frances (G) Martha Col. W.R.U.
Grime, Herschel (R-2) West Tech.
Hindman, Carrie M. (G)
Hudgeon, Nellie (G) Al. Hamilton Jr.
Jacobs, J. M. (G) Glenville
Jenney, Blanche (R-2) J. F. Rhodes
Johnson, Myrtle L. (W) Central Jr.
Kerr, Geo. P. (W) Lincoln
Lederer, Cora (R-4) Central
Lindesmith, W. B. (W) W. Wright Jr.
Lindsey, Mabel (W) Benj. Franklin
Loomis, Elisha (G) West High (retired)
McCarthy, Anne (G) West High
MacLearie, J. A. (W) Audubon
Miller, A. Brown (B-7) Fairmount Jr.
Mull, Mrs. Jane Weeks (W) Central Jr.
Nace, Edwin A. (G) South High
Neidhardt, Nina M. (W) John Adams
Nelson, Gilbert D. (G) Lincoln High
Parker, L. D. (R-2) Collinwood
Pitcher, Wilimina E. (R-2) Rawlings
Rauch, Annamary (R-2) Willson Jr.
Raush, Caroline (G) Mather Col W.R.U.
Real, Elesa B. (G) Laurel
Revesz, Lillian M. (G) Willson Jr.
Ridinger, Martha (G) Willson Jr.
Rush, Jesse J. (R-4) West High
Sampson, Helen W. (G) East Tech.
Seasholes, H. C. (W) John Adams
Selover, Hattie (R-4) Shaker Hts.
Simon, W. G. (R-4) West. R.U.
Spiers, Mrs. Nelle R. (G) Empire Jr.

Stokes, Mary (G) Laurel
Thomas, Eliz. Jane (R-3) Pat. Henry
Tomlin, Maude S. (R-2) Laurel
Tremper, Cyrus B. (R-3) East Tech.
Wallis, Edith M. (R-2) Shaker Jr.
Werremeyer, D. W. (R-5) West Tech.
Woodhouse, Kath. (G) Howells Jr.
Wright, S. M. (G) Al. Hamilton
Wyman, Carl E. (W) Audubon Jr.

Columbus
Fawcett, Harold P. (R-2) Univ. High
Griffith, Albert J. (W) Everett Jr.
Slager, Foreman W. (W) Pilgrim Jr.

Findley
Robbins, C. A. (W) High Sch.

Lakewood
Blackburn, Elta M. (G) Emerson Jr.
Needham, Grace (R-2) Emerson Jr.
Rice, Martha (G) Emerson Jr.
Rush, Abby (G) Emerson Jr.

Lorain
Simpson, Helen D. (R-6) High Sch.

Maple Heights
Wiltshire, Bernice (R-3) High Sch.

Oberlin
Cairns, W. D. (R-4) Oberlin College

Oxford
Christofferson, H. C. (R-6) Miami Univ.

Parma
Manhart, Lewis F. (G) High School

Ravenna
Collins, Hazel (R-3) City High

Salem
Douglass, Hazel (G) High Sch.

Sandusky
Denham, Elsie B. (R-2) High Sch.

Shaker Heights
Bowen, Alma (R-3) High Sch.
Miller, F. Brooks (B-7) Jr. High

South Euclid
Sister Mary Mercedes (G) Notre Dame Col.

Toledo
Refior, Sophia (W) Scott High

Vermilion
Gebhardt, Willis (R-2) High Sch.

Youngstown
Craver, Mary (W) Rayen High

*Oklahoma* (1)

Norman
Hassler, J. O. (B-7) Univ. of Okla.

*Pennsylvania* (6)

Greensburg
Silvis, Miss L. W. (W) Harrold Jr.

New Castle
McCune, Mrs. Sara S. (R-2) Sr. High

Philadelphia
Darnell, Alice H. (R-2) Germantown Friends
Stokes, C. N. (B-10) Temple Univ.

Sugar Grove
Horak, Milton (W) Voc. Sch.
Horak, Gladys (G)

*South Carolina* (1)

Columbia
Bremen, Francenia (G) High Sch.

*Tennessee* (1)

Nashville
Wren, Frank L. (R-2) Peabody Col.

*Virginia* (1)

Charlottesville
Whittlock, W. Carl (G) Allyn & Bacon

*Wisconsin* (4)

Madison
Hartung, M. L. (R-2) Univ. High

Milwaukee
Turner, Leonard S. (R-2) Shorewood

Racine
Potter, Mary A. (B-12) Supervisor

Whitewater
Bigelow, Oramel (R-3) Tea. Col.

ATTENDANCE BY STATES

| | G. | W. | R. | B. | Total |
|---|---|---|---|---|---|
| 1. Colorado | 0 | 1 | 0 | 1 | 2 |
| 2. Dist. of Columbia | 0 | 1 | 1 | 0 | 2 |
| 3. Georgia | 0 | 1 | 0 | 0 | 1 |
| 4. Illinois | 1 | 4 | 2 | 5 | 12 |
| 5. Indiana | 1 | 2 | 3 | 1 | 7 |
| 6. Iowa | 0 | 0 | 2 | 0 | 2 |
| 7. Maryland | 0 | 0 | 2 | 0 | 2 |
| 8. Massachusetts | 0 | 0 | 3 | 1 | 4 |
| 9. Michigan | 0 | 3 | 7 | 3 | 13 |
| 10. Minnesota | 0 | 0 | 1 | 0 | 1 |
| 11. Missouri | 0 | 0 | 3 | 0 | 3 |
| 12. Nebraska | 0 | 0 | 1 | 0 | 1 |
| 13. New York | 3 | 4 | 7 | 3 | 17 |
| 14. Ohio | 31 | 20 | 34 | 4 | 89 |
| 15. Oklahoma | 0 | 0 | 0 | 1 | 1 |
| 16. Pennsylvania | 1 | 2 | 2 | 1 | 6 |
| 17. South Carolina | 1 | 0 | 0 | 0 | 1 |
| 18. Tennessee | 0 | 0 | 1 | 0 | 1 |
| 19. Virginia | 1 | 0 | 0 | 0 | 1 |
| 20. Wisconsin | 0 | 0 | 3 | 1 | 4 |
| | 39 | 38 | 72 | 21 | 170 |

CODE: G = Visitor or guest.
W = Member attending his first annual meeting.
R = Member attending 2–6 annual meetings.
B = Member attending seven or more annual meetings.
B-14 = Member attending 14 annual meetings.

REPORT OF THE TREASURER FOR THE YEAR
FEB. 1, 1933–FEB. 1, 1934

| | | | |
|---|---|---|---|
| Balance on hand at beginning of year | | | |
| Union National Bank of Macomb, Ill. | | $1882.26 | |
| Savings Bank Deposit | | 412.19 | |
| New York Telephone Bond, 4.5, 1939 | | 982.50 | |
| Commonwealth Edison Bond, 5, 1953 | | 935.00 | |
| | | | $4211.95 |
| Receipts for the year | | | |
| W. D. Reeve; Yearbooks | $180.25 | | |
| MATHEMATICS TEACHER | 898.49 | | |
| | | $1078.74 | |
| Bureau of Publications | | | |
| Yearbooks ($369.25 credit towards 9th Yearbook) | | | |
| Interest on Bonds | | 94.00 | |
| Interest on Savings | | 15.82 | $1188.56 |
| | | | $5400.51 |

| | | | | |
|---|---|---|---|---|
| Expenditures for the year | | | | |
| Annual Meeting | | | | |
| Directors' Expenses | | $757.34 | | |
| Speakers | | 212.00 | | |
| Local Committee | | 24.00 | | |
| Printing | | 6.86 | | |
| Directors' Tea | | 5.65 | | |
| | | | $1005.85 | |
| Officers and Committees | | | | |
| Sec.-Treas. Office | | | | |
| Stationery | $ 14.80 | | | |
| Supplies, etc. | 50.46 | | | |
| Tax on checks | .74 | | | |
| Secretarial Service | 450.00 | 516.00 | | |
| Pres. Office (Boston Meeting) | | 35.00 | | |
| Chicago Meeting (Summer) | | 81.78 | | |
| Geometry Committee | | 52.08 | | |
| | | | 684.81 | |
| Total Expenditures for the year | | | | $1690.66 |
| Balance on Hand, February 1, 1934 | | | | $3709.85 |

| | | |
|---|---|---|
| BALANCE SHEET, FEB. 1, | 1934 | (for 1933) |
| ASSETS—Commercial Bank Deposit | 1270.34 | (1882.26) |
| New York Telephone Bond | 982.50 | ( 982.50) |
| Commonwealth Edison Bond | 935.00 | ( 935.00) |
| Savings Bank Deposit | 522.01 | ( 412.19) |
| Total Assets | 3709.85 | (4211.95) |

LIABILITIES—None

(Signed) EDWIN W. SCHREIBER, *Treasurer*

The above account has been audited and found correct.

(Signed) VERA SANFORD

## SALE ON YEARBOOKS

**The National Council Yearbooks—two to nine inclusive—may now be obtained postpaid for $11.00 from *The Bureau of Publications, Teachers College, Columbia University,* 525 West 120th Street, New York, N. Y. See first page of this issue for particulars.**

# Analysis Is Not Enough

(*A treatise on the teaching of geometry in the light of Gestalt Psychology*)

*By* ALMA M. FABRICIUS

*Seward Park High School, New York City*

EVER SINCE the explosion of the theory that the faculty of thinking could be developed and strengthened by exercise in thinking regardless of the nature of the subject matter involved, geometry as a universally required subject in the high schools of America has been on the defensive. And, when we consider the large number of failures in the subject, in the light of the educational theory that a child learns only through the encouragement of success, it becomes seriously doubtful whether geometry should be retained as a compulsory subject in the high school.

On the other hand, we are deterred from such a ruthless conclusion when we consider the importance of geometric concepts in the daily life of modern society, and the great satisfaction and mental pleasure derived from the study of the subject by those who do succeed in it.

Mr. William Betz, in an article on "Intuitive Geometry" in the Eighth Year Book of the National Council of the Teachers of Mathematics states the practical case rather graphically as follows:

> We are living in a world which is incurably mathematical. Number and form accompany us wherever we go. We cannot make or manufacture the simplest object without giving due consideration to its shape, its size, and the correct position of its parts. And as problems of construction become more complex, correspondingly greater demands are made on exact geometric knowledge. Thus, a skyscraper or an automobile, or a bridge, or a tunnel represents a veritable symphony of applied geometry.
>
> Hundreds of trades depend for their very existence on precise measurements, on blue prints and scale drawings. Maps, charts, and graphs are the very warp of modern travel and commerce, and the calendar we use is made possible only by a continuous survey of the heavens. In short, what subject can boast of a more all pervasive relation to modern life?

Considerations like the above prompt us to examine geometry as it is taught today to see whether its rich field of subject matter

is being presented to the student in the best possible way from a psychological point of view. A brief survey of the history of the teaching of the subject may help us locate its defects and perhaps point the way to the possibilities of improvement in the future.

Geometry has the misfortune of having been one of the first subjects to reach a mature stage. As early as 300 B.C., long before any of our modern theories of psychology were even dreamed of, it had already become fixed in sequence of subject matter. The sequence that Euclid gave it was a purely logical sequence, and was intended for mature students of logic. As a carrier of logical form, rather than of subject matter, it was taught in the medieval universities. From the medieval universities it came down into the secondary schools of Europe still retaining its place as a model of systematic thinking. From the European secondary schools it came to the American high schools.

In its progress through the secondary schools of Europe and America, geometry ran abreast of all the educational philosophies and psychologies that held sway during the different periods of its existence; and it will be interesting to note the mark that each of these theories left upon it.

In the days of the faculty psychologists, who held that reasoning begot reasoning, geometry was a mere matter of following the proofs that were set down in detail in the text-book. The method employed by the instructor in these days was largely the socratic and lecture method, and the text-books called for practically no original work on the part of the student.

The faculty psychologists were driven off the stage by the associationists who held that the mind was not divided up into compartments which could be strengthened by exercise. Learning, according to this school, consisted in forming bonds and the teacher's function was to produce the most favorable setting for the formation of those bonds. The most notable contribution of this school was Herbart's "Five Formal Steps of a Recitation" which are *preparation*, *presentation*, *comparison*, *generalization* and *application*.

The five formal steps were a decided advance over the old method of teaching, but, still, in practice, left much to be desired. As a rule, the first four steps were carried on by the teacher, with responses from as many pupils as could be stimulated to respond in the limited time allotted to this phase of the recitation. The stu-

dent activity came in the application. Geometry text-books under the Herbartian influence began to include exercises which made use of the propositions in their solution. These exercises consisted of proving exercises, numerical work and constructions, with by far the greatest emphasis on the first of the three. In other words, the pupils, in their application, were beginning to associate exercises with the propositions as a core, and some of the pupils were beginning to see the logical connection between the propositions as a result of the teacher's activity and their responses. Undoubtedly this led to a better understanding of the work on the part of all students.

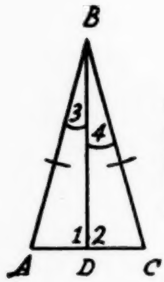
The next notable change in geometry teaching we find in response to Dewey's precept, "Learning is doing". Arthur Schultze in his text on the *Teaching of Mathematics in the Secondary Schools* published in 1916, states that only the original exercises justified the teaching of geometry. The true end of mathematics is power, and not knowledge of the facts imparted. The proving of propositions must be considered secondary. The work with original exercises is primary.

This shift in emphasis was exhilarating, but left the teacher somewhat helpless. It is one thing to say that the original exercise is all important and another thing to get the student to do the original exercise. In other words, the problem was, how to prevent the student from memorizing the original exercise which someone else had worked out for him, just as before he had memorized the propositions the teacher or text-book had developed for him.

In an effort to solve this difficulty, there arose a great faith in what was called "analysis" among the mathematical educators. Young teachers were told that if they could only get their students to analyse, the battle would be won. The thought of a student in the process of analyzing a problem is as follows:

"I can prove $X$ if I can prove $Y$,
and I can prove $Y$ if I can prove $A$,
but I can prove $A$ if $B$ is true.
$B$ is true; hence I can prove $A$." X

To illustrate from a particular problem, let us take the proof of the exercise, "The bisector of the vertex angle of an isosceles triangle is perpendicular to the base."



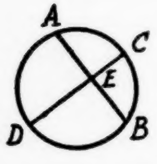

The student analyzing, after writing down hypothesis and conclusion, and marking the same on his diagram, is supposed to go through the following reasoning: "I can prove $BD \perp AC$ if I can prove $\angle 1 = \angle 2$, which I can prove if I can prove $\Delta ABD \cong \Delta DBC$, which I can prove if I have $s.a.s. = s.a.s.$ But I have $s.a.s. = s.a.s.$ and therefore I can prove the exercise.

The student, as is obvious from the above example, is encouraged to move backwards from his conclusion to what is given, the assumption being that he can move backwards with greater ease and assurance and with greater hope of success than would be possible if he moved forward toward his goal.

Where the exercises to be proved are of the simplest type, this little device works beautifully, especially where the choice of steps to take is very limited. At the time the student proves the exercise just described, he has had only one way of proving lines perpendicular, practically only one method of proving angles equal in the type of figure before him, and possibly only two or three ways of proving triangles congruent. Hence the danger of his slipping off into a blind alley is very small.

But teachers find that as geometry becomes more involved and the possible number of ways of taking the next step more and more numerous, analysis as a sole method of procedure, becomes less and less satisfactory, and less and less sure of success.

Let us take a problem which occurs at about the end of the first term of geometry.




Given: $AE = EC$

Prove: $AB = DC$

The analysis for this problem would read as follows: "I can prove $AB = DC$ by one of many ways: by proving $\overset{\frown}{AB} = \overset{\frown}{DC}$; or by proving $DE = EB$ and then adding equals to equals; or possibly by proving triangle $ABC$ congruent to triangle $ACD$. There is also a possibility of my proving triangle $AED$ congruent to $CEB$, and

so proving $DE=EB$. Which of these four possible steps I shall take will depend entirely upon the possibilities that flow from the facts given. In other words, before I can take the first step backward, I must trace my entire route forward. This is reminiscent of a comment made by one of my students, when as an unsophisticated novice, I was trying to impress upon her the magic of analysis, "I can write the analysis if I know how to do the example but not otherwise."

Professor Frank Charles Touton, in a survey of the solutions written on the passing papers of a representative number of high schools of the state in one of the statewide Regents examinations in Plane Geometry makes the following comment on what he found:

> The records made show that pupils do identify more or less carefully the given elements, yet with those who employ long methods, the elements in the desired outcome seem over potent in determining the procedure—a procedure which too often involves the drawing of unnecessary construction lines. The desired outcome should be a potent element in the solution, but its potency should not be so great as to determine the method of procedure more or less independent of the given data.

This passage may be considered as a commentary on the dangers attendant upon too zealous a pursuit of the method of analysis, to the exclusion of synthesis in attacking a problem. Analysis is obviously not the "open sesame" that its sponsors claim it to be. Its great popularity at the present time is probably due to the tendency the human race has of swinging from one extreme to the other, and to avoid the less sensational middle course, which, more often than not, is the path of wisdom.

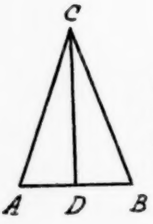
Solving a problem is somewhat like building a tunnel under a river. It is no greater wisdom to start exclusively on the one end than on the other. The project must be started at both ends, and work at each end directed toward meeting the work at the other end. In problem solving we must train the student to move in both directions until the two approaches result in what the gestalt psychologists call a "closure" or "insight."

Under the influence of analysis, text-books published within the last few years have shown a tendency to include "methods" in their context. After a proposition which is specially useful in proving some type of fact, a note will appear, labelled "Method," stating how the proposition is to be used. At the end of each chapter we find lists of methods which grow increasingly voluminous as

the chapters advance. Thus on page 94 of a book of 350 pages, published in 1933 the author gives the following list of methods for proving angles equal:

a. If they are right angles.
b. If they are complements of the same or equal angle.
c. If they are supplements of the same or equal angles.
d. If they are vertical angles.
e. If they are angles opposite equal sides in a triangle.
f. If they are corresponding angles of congruent triangles.
g. If they are alternate interior or corresponding angles of parallel lines.
h. If their sides are parallel (or perpendicular), right side to right side and left side to left side.
i. If they are third angles of two triangles having two angles of one equal respectively to two angles of the other.
j. By the use of axioms.
k. By superposition.

A little earlier in this text-book, the author gives his readers instructions which throw some light on how he intended such lists to be used. He lays down the following advice in connection with deriving the proof for the theorem, "The base angles of an isosceles triangle are equal."




In selecting a method of proof you should first search the axioms, postulates, definitions, and theorems and corollaries already proved to find all known methods which might be used to prove the theorem. Then select the method most suitable to your proof.

In this theorem, you wish to prove that two angles are equal. So you recall all the methods you have had of proving angles equal. These methods are: the first eight axioms, Post. 9, pp. 38, 39, 41, 42 and Post. 17. You then study these methods and determine which one can be used. From a study of the axioms you find no relations by which you can prove $\angle A = \angle B$. You next try Post. 9. Since $\measuredangle A$ and $B$ are not right angles, Post. 9 does not apply. You refer to pp. 38, 39, 40, 41 and 42. Since $\measuredangle A$ and $B$ are neither vertical angles nor complements or supplements of the same or equal angles, these theorems do not apply. Post. 17 states that "correspond-

ing parts of congruent triangles are equal." Possibly you may be able to prove that $\angle A = \angle B$ if you can draw a line dividing $\triangle ABC$ into two congruent triangles. . . .

At best the procedure outlined leads to stilted and uneconomical thinking, and does not at all resemble the way a child would naturally approach the solution. Had this problem been presented to the student at a time when he had already become familiar with the general pattern as a result of experiences with similar diagrams that included the necessary line, he would have seized upon the correct method at once without the tedium of the recommended series of trial and error attempts, and the immediate insight would have led to better learning than the rambling movements suggested above.

This is a good example of the danger of placing too much emphasis on the conclusion. The psychological approach to this problem is one that attaches meanings to the diagram, rather than one that associates a tiresome list of methods with the conclusion.

This brings us to the point of this article. It is not enough to so organize the subject matter of geometry as to make the backward movement or analysis easier. It is equally important to give impetus to the forward movement from the facts and the diagram, which is commonly called the synthetic approach to the problem.

Here is where the traditional text-book is woefully deficient. The text-book from which I have quoted gives practically no stimulus toward a forward moving process in the attack upon a problem, except to mark an occasional diagram. Such marking is done without comment and certainly without emphasis.

Students should be made to realize that like men in a chess game, every basic figure has its special powers and possibilities of movement. In an isosceles triangle, one can move from two equal sides to two equal angles, and vice versa. In this triangle, the bisector of the vertex angle, and the median and altitude to the base, all three coincide and are, therefore, interchangeable. The medians, bisectors and altitudes to the legs are equal pairs, and cut off equal segments on the legs.

Among the quadrilaterals, the kite-figure leads to the bisection of angles and lines, and to perpendiculars. A rhombus is even more powerful in the same directions because all of its angles are bisected and both of its diagonals. In a parallelogram, all four angles are related. Hence one can move from one angle diagonally across to the opposite angle which is equal to it, or to the consecu-

tive angle which is supplementary to it. It also enables one to get from one side to an equal side opposite it, and, in addition, produces the phenomenon of two lines bisecting each other.

In the circle we have the inscribed quadrilateral, which, while not as powerful as the parallelogram, nevertheless enables one to move from one angle diagonally to a supplementary angle. A diameter in a circle leads to equal semi-circles or to right angles either inscribed in those semi-circles or with a tangent meeting it at one end. A tangent may lead to a tangent-chord angle or a right angle, depending upon whether the line meeting it is an ordinary chord or a diameter.

Furthermore, there are concepts other than the basic figures which may lead to useful organizations. A mid-point in the facts may lead to two equal halves of the same line or to a median which gives two equivalent triangles. If it is the midpoint of two intersecting lines, it may produce a parallelogram with its many attendant consequences. Two mid-points may lead to two equal halves of different lines, or to a line parallel to the third side of a triangle and equal to half of it. Four mid-points may lead to a parallelogram.

The proper combination of equal sides and angles in two triangles will produce congruence which results in additional pairs of equal sides and angles. A proper combination of equal angles, or equal angles and ratios, or equal ratios alone, lead to similar triangles with their consequent new equal ratios and equal products.

Such are the possibilities of meanings that may be attached to the given facts to stimulate synthesis. It is elementary that in order to make these figures and concepts dynamic with meaning there must be a conscious organization of experiences to bring out these characteristics. There must be a conscious organization of exercises around figures and concepts to bring out their possibilities.

The traditional text-book fails in this respect. Its exercises are grouped only around propositions. The ideal text-book should have both types of organizations, according to propositions to help analysis and according to basic figures to stimulate synthesis.

# J. O. Hassler

THE NEW PRESIDENT of the National Council of Teachers of Mathematics is completing his twenty-ninth year of teaching. He began teaching in the country schools of Missouri at the age of seventeen. Since graduating from college he has done ten years of high school and seventeen years of college and university teaching. He taught mathematics and history in the Douglas (Ariz.) High School, 1907–11; mathematics in the University of Kansas, 1911–12; mathematics in the Englewood (Chicago) High School, 1912–17; mathematics and astronomy in the Crane Technical High School and Junior College, 1917–20; mathematics and astronomy since 1920 in the University of Oklahoma, where he is now professor of mathematics and astronomy.

His education was begun in the ungraded country schools of Missouri and continued in an obscure private academy and William Jewell College, where he received the A.B. degree in 1907. He did graduate work at the universities of Kansas and Chicago, receiving the degree Master of Science at the latter institution in 1913 and Doctor of Philosophy in 1915. At Chicago in 1914 he was elected to membership in Sigma Xi. He is also a member of the American Mathematical Society and a Fellow of the American Association for the Advancement of Science.

For three years, 1916–19, he was editor of the Problem Department, *School Science and Mathematics*. He is the author of *Plane Geometry, Solid Geometry;* co-author (with R. R. Smith) of *The Teaching of Secondary Mathematics;* co-author (with W. L. Vosburgh) of *Junior High School Mathematics*, Books I, II, III; besides articles in scientific and pedagogical journals on mathematics and the teaching of mathematics.

He went to Oklahoma during the first year of the existence of the National Council and immediately began working in its behalf. At the meeting of the Mathematics Section of the Oklahoma Education Association held that year he asked permission of the chairman to speak in behalf of the Council and from an attendance of fifty secured seven members. He has never missed a meeting since and has never failed to present, or have presented, a plea for memberships. The Mathematics Section is now a branch of the Council.

# NEWS NOTES

## Recommendations Concerning Demonstrative Geometry and Advanced Mathematics from the Association of Teachers of Mathematics in New England

At a Meeting held in Cambridge, January 20, 1934, at which 80 members were present, the Association passed the following votes concerning geometry, for communication to the Committee on Geometry of the National Council of Teachers of Mathematics and to the Commission on Mathematics of the College Entrance Examination Board. These votes grew out of a series of resolutions which were framed by two committees of this Association, one committee representing the eastern part of New England, the other the Connecticut Valley Branch of this Association.

1. The requirement in geometry, as officially defined, should neither explicitly nor by implication suggest any teaching order, nor should full credit on examination be denied on the ground that any one theorem must necessarily precede some other theorem. (Yea 31, Nay 5.)

2. Trivial demonstrations can be excluded by some such regulation as the following: "When a theorem is proved merely by showing it to be an obvious corollary of another theorem, the proof of that other theorem is required." (Yea 40, Nay 2.)

3. The requirement should be stated in the form of a list of theorems arranged by topics but without implying any definite teaching order. (Yea 40, Nay 1.) There should be starred propositions if the examination is to cover book theorems. (Yea 15, Nay 10.)

4. The use of algebra and of trigonometric ratios should be encouraged for demonstration as well as for problems of computation. For example, the statement and the proof of the theorems referring to the third side of a triangle when the other two sides and their included angle are given become in algebraic form not only simpler but more comprehensive. (Algebra: Yea 45, Nay 0. Trigonometry: Yea 30, Nay 2.)

5a. It should no longer be permissible to ignore incommensurable magnitudes; some form of the theory of limits, or some fairly adequate treatment of irrational numbers should be required in geometry. (Yea 35, Nay 4.)

5b. This requirement should be enforced by *questions set on examination* from time to time. (Yea 10, Nay 30.)

6. Some effort should be made to bring to the attention of pupils the invaluable logical content of geometry, and thus make that content more readily available in matters not connected with geometry. (Yea 35, Nay 0.)

7. Pupils should be encouraged to quote a theorem in a general and comprehensive form, rather than in a special case; for example C47* instead of C48 or C49, and C30* instead of C42. The notation C47*, etc., refers to Document 108. (Yea 20, Nay 8.)

8. Teachers should be advised not to require extreme formality in demonstration, or to insist on such rigmaroles as "AB = AB by identity," or on reference to axioms of algebra which are never referred to in their regular algebraic work. (Yea 40, Nay 2.)

9. No formal distinction should be

made for examination purposes between "book theorems" and "originals." The purpose of this change is to give greater freedom in teaching. (Yea 40, Nay 3.)

10. Theorems which require superposition for their proof may be postulated. (Yea 35, Nay 4.)

11. The usual list of postulates may be extended. (Yea 25, Nay 2.)

12. Some terms ostensibly defined ought to be taken as undefined. (Yea 15, Nay 5.)

13. It is recommended that much of the factual content of plane and solid geometry be taught as informal geometry in grades 7, 8, 9. (Yea 25, Nay 1.)

14. Some pertinent ideas from solid geometry ought to be exhibited concurrently with a first course in demonstrative geometry. (Yea 30, Nay 0.)

15. Geometry should be taught so as to obtain transfer of logical training to non-geometric situations. (Yea 15, Nay 0.)

16. The logical structure of geometry should be emphasized. (Yea 20, Nay 1.)

At a meeting in Cambridge on March 10, 1934, at which 48 members were present, the Association passed the following votes concerning advanced mathematics, for communication to the Commission on Mathematics of the College Entrance Examination Board.

1. A full year course consisting of trigonometry, solid geometry, and advanced algebra is to be preferred to the present practice of offering half-year courses in only two or these three subjects. (Only one dissenting vote.)

2. Several votes were taken to determine the sense of the meeting concerning the amount of time to be devoted to each subject in such a course.

(a) With respect to trigonometry, only 3 were opposed to giving less than a half-year course in addition to the instruction in trigonometry now commonly included in elementary algebra; 25 would accept a reduction to 40 per cent of the year; and 16 of these latter felt that 33 per cent of the year was sufficient for the trigonometry.

(b) With respect to solid geometry, a few desired at least 40 per cent of the year; 5 would not oppose a reduction to 25 per cent of the year; but the majority wanted at least 33 per cent of the year for this subject.

(c) With respect to advanced algebra, the theory of equations was unanimously regarded as the most important topic: next in importance came permutations, combinations, and probability, with pertinent reference to the binomial theorem and compound interest. With 40 per cent of the year allotted to trigonometry, and 33 per cent to solid geometry, there remains 27 per cent of the year for these topics in algebra.

3. (a) Only 6, of whom 5 were secondary school teachers, favored mention of the derivative in connection with the theory of equations.

(b) A committee of the Connecticut Valley Branch of this Association favored a year course in advanced mathematics, 40 per cent of which should be devoted to trigonometry, and about 20 per cent to solid geometry, making possible an allotment of 30 per cent of the year to algebra, including certain topics from analytic geometry, and an allotment of 10 per cent to the study of slopes and areas by means of differentiation and integration.

4. The College Entrance Examination Board is requested to consider the desirability of replacing the present examinations in advanced mathematics by a single examination on an undivided year course in advanced mathematics,

the paper to be so constructed that questions must be answered in each subject while permitting sufficient option to allow teachers some latitude in selecting material for instruction.

### Chicago Mathematics Club Celebrates Twentieth Anniverary

On January 19, 1934, the Men's Mathematics Club of Chicago and Metropolitan Area celebrated the Twentieth Anniversary of the founding of the Club. The fifty-five men present heard the recounting of past experiences, and of early efforts at charting the course in mathematics, and stories from and about the men that have become National Figures in the Field of Mathematics.

After a few remarks appropriate to the occasion, the President of the Club, Mr. Francis W. Runge of Evanston Illinois Township High School, turned the gavel over to Mr. C. M. Austin of Oak Park High School who remained Chairman for the evening. Mr. Austin was the first President of the Men's Mathematics Club of Chicago and Metropolitan Area during the period 1913–16. He was also the first President of the National Council of Teachers of Mathematics.

The Chairman's preliminary statements touched upon some early experiences of the Club and the nature of the meetings. He then directed the secretary to read the letters that he had received from members who were unable to attend. Professor E. R. Breslich of the University of Chicago was unable to attend due to an important conference out of town that necessitated his presence. Mr. H. E. Cobb, formerly of Lewis Institute, but now retired near the "Rockbound Coast of Maine," sent his greetings. Mr. Alfred Davis of Soldan High School, St. Louis, Missouri, regretted his inability to be present and wrote of a victory for required mathematics in the St. Louis High Schools.

We were greeted, via letter, from the State Normal School at California, Pennsylvania, by J. A. Foberg, who expressed a desire to be present at our party, Mr. G. A. Harper, who owns and administers the Southern Arizona School for Boys, wrote us from Tuscon. He recalled past pleasantries and suggested that members of our Club who were looking for REAL work should open a private school in their old age. Mr. J. O. Hassler, Professor of Mathematics at the University of Oklahoma, desired to be present but business and distance prevented. Professor W. D. Reeve of Teachers College, sent warm greetings, recounted some pleasant experiences and wished our party success. From the University of Michigan we received a long and pleasant letter in which Professor Raleigh Schorling told some of his experiences while a member of our Club.

The reading of the letters was followed by speeches, poetry, stories, and jokes. These were related by past Presidents of the Club and by the "Old Timers" present. The verbal contributions of the many speakers were excellent but the stories told by Mr. E. W. Owen of Oak Park High School, and Mr. Marx Holt, Principal of a Chicago High School "Of Peaches of Georgia and points South," stand out as entertainingly potent. It is interesting to note that only two of the past Presidents were not in attendance. These were Professor John R. Clark of Columbia University, and Mr. Olice Winter, Principal of Lake View High School, Chicago. Although they were not present, nevertheless, members who knew them well and knew of their work presented them to us in spirit.

The names of the past Presidents and the years during which they held the office follow: C. M. Austin, 1913–16; J. R. Clark, 1916–18; W. W. Gorsline, 1918–20; M. J. Newell, 1920–21; H. C. Wright, 1921–22; E. W. Owen, 1922–23; Olice Winter, 1923–24; E. W. Schreiber, 1924–25; Marx Holt, 1925–26; O. M. Miller, 1926–27; E. S. Leach, 1927–28; J. T. Johnson, 1928–29; W. H. Clark, 1929–30; Charles Leckrone, 1930–31; E. C. Hinkle, 1931–32; W. S. Pope, 1932–33. The present incumbent is Mr. Francis W. Runge.

During the first year (1913–14) the following men organized and became affiliated with the Club: C. M. Austin, E. R. Breslich, J. R. Clark, H. E. Cobb, S. J. Conner, W. W. Gorsline, G. A. Harper, Lawrence Irwin,* C. E. Jenkins, F. A. Kahler, O. M. Miller, James Millis,* G. W. Myers,* M. J. Newell, W. D. Reeve, Harold O. Rugg, F. W. Runge, Raleigh Schorling, H. E. Slaught, W. A. Snyder, Olice Winter, and H. C. Wright. (* Now deceased.)

In the years that followed other men became affiliated with the Club. The following list presents those that joined previous to the 1918–19 season: A. M. Allison, A. W. Cavanaugh, M. W. Coultrap, Alfred Davis, J. A. Foberg, E. C. Hinkle, Marx Holt, J. T. Johnson, J. O. Hassler, J. M. Kinney, C. E. Kitch, Butler Laughlin, Charles Leckrone, D. W. Merrill, J. A. Nyberg, E. W. Owen, J. C. Piety, E. W. Schreiber, G. C. Staley, and G. G. Taylor.

A perusal of the foregoing list of names coupled with a knowledge of prominent mathematicians should indicate clearly to the reader the early field of professional operation of many.

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THE SPRING MEETING of the Colorado Chapter of the National Council of Teachers of Mathematics was held in Denver, March 24. The subject for discussion was "The Present Critical Situation in Secondary Mathematics."

Several college professors talked and two men from the smaller districts. The idea was to bring all those interested in the teaching of mathematics into closer relation. The colleges told us of some places where we might strengthen our teaching. Those who talked from the junior and senior high schools showed what they were doing to improve the teaching.

Professor G. W. Finley of State Teachers College showed us that all life is based on mathematics and that it is impossible to leave mathematics out of the course of study.

Carl Melzer, of Breckenridge, Colorado talked briefly on showing pupils why a rule is made instead of simply teaching the rule.

A. B. Ewer of Johnstown, Colorado gave us some interesting ideas on how to combine geometry and art. His work was very clever. He showed us that in coloring the designs made the pupils could not help but get an idea of areas, arcs, circles, polygons, etc. Those of us who listened were fascinated and I'm sure his pupils must feel the same way.

Dr. Stearns of the University of Denver gave us a very encouraging talk. He showed us that mathematics is a universal language. Most sciences can be interpreted by means of the language of mathematics. He made us feel that the work of the Secondary School was to see that this language is so well learned that it may be put to use when the pupils go into the sciences.

Luncheon was served at the hotel and the meeting resumed in the afternoon. This part of the meeting was devoted to the report of Alfhild Alenius who was our delegate to the National Council in February.

Dr. Aubrey Kemper of University of Colorado gave us a brief report on the

bulletin of the American Association of University Professors.

Mary S. Sabin, retired Denver teacher, and Professor Ira De Long, retired University of Colorado Professor, gave us a few words of greeting. Miss Odell of Denver gave us some of the work of the committee on Geometry.

President Professor S. L. McDonald, Colorado Agricultural College

Vice Pres. Mrs. Ruby S. Flannery, Denver

Sec.-Treas. H. W. Charlesworth, Denver

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THE ANNUAL MEETING of the Illinois Division of The National Council of Teachers of Mathematics was held in room 300, Mathematics Building, University of Illinois on November 24, 1933, Mr. J. T., Johnson, Chicago Normal presiding.

Morning Session: 9:15.

"The Results of Teaching Geometry by Flash Cards" Miss Gertrude Anthony, Oak Park and River Forest H. S.

"A Comparative Study of Long and Short Periods in Algebra" Mr. E. G. Hexter, Belleville Township H. S.

Mr. E. W. Schreiber spoke to the group concerning membership in the National Council and subscription to the Mathematics Teacher.

Mr. C. M. Austin distributed questionnaires for the committee on Integration.

Afternoon Session: 2:10.

Short business meeting.

"A Student in England," Dr. Echo D. Pepper, University of Illinois.

"On The History of the Metric System" an illustrated lecture Mr. E. W. Schreiber, State Teachers College, Macomb.

Dr. Lytle recommended membership in the National Council and ownership of the Year Books to high school teachers of mathematics.

Committee for meeting in 1934:

Mr. Wm. Clark, Champaign High School, Chairman

Miss Martha Hildebrandt, Proviso High School, Vice Chairman.

Mr. E. G. Hexter, Belleville Township H. S. Secretary.

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